Library Progress International *Print version ISSN 0970 1052*
Vol.45, No.2, Jul-Dec 2025: P.410-413 *Online version ISSN 2320 317X*

Original Article *Available online at www.bpasjournals.com*

Blood Flow Analysis in the Stenosed Artery using Galerkin Approach

Neetu Srivastava[1*] and Asha K N[2]

[1] Associate Professor, Department of Mathematics, Amrita Vishwa Vidyapeetham, Bengaluru, India.
[2] Assistant Professor, Department of Mathematics, CMR Institute of Technology, Bengaluru, India.
[*] Corresponding author, Email address: asha.kn@cmrit.ac.in

How to cite this article: Neetu Srivastava and Asha K N (2025). Blood Flow Analysis in the Stenosed Artery using Galerkin Approach. *Library Progress International,* 45(2), 410-413

Abstract: An exploration has been done using the Galerkin method to examine the effects of a magnetic field on steady blood flow in an inclined, tapered artery with stenosis. The flow is assumed to be time-independent and axisymmetric. The physical scenario is modeled through mathematical equations, accompanied by suitable boundary conditions aligned with magnetohydrodynamic theory. Various flow characteristics within the stenosed artery have been computed and illustrated graphically for different parameter values. These findings underscore the importance of considering arterial geometry and magnetic effects in hemodynamic modeling for improved diagnosis and treatment of cardiovascular conditions.

Keywords: Galerkin Method, Magnetohydrodynamics, Tapering

Introduction

The investigation of hemodynamics in arteries with stenosis has drawn notable focus as a result of its direct implications for cardiovascular health. Stenosis, a condition characterized by the narrowing of blood vessels, disrupts normal hemodynamics and can lead to serious complications such as heart attacks and strokes. Understanding the flow behavior in stenosed arteries is essential for both diagnostic and therapeutic advancements in medical science.

In recent years, the application of magnetohydrodynamics (MHD) in biomedical engineering has set the stage for controlling and analyzing blood flow, particularly in the presence of magnetic fields. Blood, being an electrically conducting fluid due to the occurence of iron-containing hemoglobin in red blood cells, interacts with applied magnetic fields in a complex manner. This interaction can influence velocity profiles, wall shear stress, and pressure gradients within the vascular system. The ability to manipulate these parameters non-invasively has important clinical relevance, especially in the design of medical devices and targeted drug delivery systems.

Arterial geometries such as tapering and inclination, commonly observed in various anatomical regions, further complicate the flow dynamics, especially when stenosis is present. Accurate modeling of such physiological conditions requires a comprehensive modeling scheme that captures geometry, fluid properties, and external influences like magnetic fields.

In this context, the present investigation employs the Galerkin method to analyze the steady, axisymmetric blood flow through an inclined and tapered artery affected by stenosis, under the influence of a transverse magnetic field. The mathematical formulation is developed in line with the governing principles of magnetohydrodynamics and incorporates realistic boundary conditions to ensure physical relevance. The resulting equations are solved to evaluate the impact of various parameters on flow characteristics, and the findings are presented graphically. The study uncovers key information about the behavior of blood flow in pathological arterial segments, potentially aiding future clinical applications.

Srivastava (2014) examined the flow behavior of blood through an inclined tapered porous artery with mild stenosis under the influence of an inclined magnetic field. The study utilized a mathematical approach to understand the interplay between inclination, tapering, and magnetic forces, concluding that these factors significantly alter the flow dynamics and pressure distribution. Roy et al. (2017) presented a computational model focusing on general blood flow through stenosed arteries. Their study emphasized the impact of stenosis severity and vessel geometry on velocity profiles and shear stress. The authors highlighted the importance of numerical simulations in capturing the complexity of real-life arterial flows. Prasad and Yasa (2020) explored the non-Newtonian nature of blood and its flow through a permeable artery with non-uniform cross-section and multiple stenoses. The analysis revealed that both permeability and multiple stenoses substantially affect the resistance to flow and the wall shear stress, offering insights into pathological conditions involving repeated arterial blockages. Haghighi et al. (2019) developed a model for micropolar blood flow through a stenosed artery, incorporating body acceleration and an external magnetic field. Their results showed that micropolar effects and body forces significantly influence rotational velocity and stress fields, which are crucial for realistic physiological modeling. In a more recent study, Prasad and Yasa (2021) investigated the flow of nanofluid through an inclined tapering artery with stenosis in a porous medium. The combined effects of magnetic field, nanoparticle concentration, and arterial inclination were shown to control the temperature and velocity fields, suggesting potential applications in hyperthermia-based treatments.

In this problem we tried studying the Srivastava's (2014) problem using Galerkin's method. We have compared results obtained in both the papers.

Mathematical Formulation

The mathematical model for the time-invariant flow in an inclined artery is expressed as:

$$\left.\begin{aligned} \rho \boldsymbol{F} &= -\boldsymbol{\nabla} p + \mu \nabla^2 \boldsymbol{V} - \mu \frac{\boldsymbol{V}}{\kappa} + \boldsymbol{JXB} \\ \nabla . \boldsymbol{V} &= 0 \end{aligned}\right\} \quad (1)$$

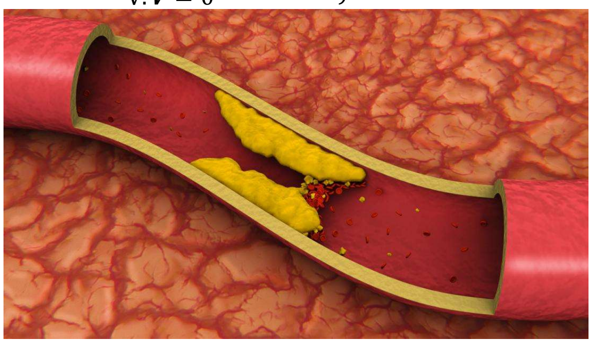

Figure1. Schematic of the diseased artery.

The geometry of stenosis is defined by:

$$h(z) = \begin{cases} l(z)[1 - \vartheta\,(b^{n-1}(z-a) - (z-a)^2)], & a \le z \le a+b \\ l(z), & Otherwise \end{cases}$$

with radial dimension of the narrowing tapered artery segment $l(z) = l + \xi z$, uniform radius of the healthy arterial segment is denoted by l_0, tapering parameter is denoted by ξ, length of the stenosis is denoted by b and n$\geq$ 2 is shape parameter. The parameter ϑ is defined by:

$$\vartheta = \frac{\delta n^{\frac{1}{n-1}}}{d_0 b^n (n-1)},$$

where δ denotes maximum height of the stenosis located at

$$z = a + \frac{b}{n^{1/(n-1)}}.$$

Conditions for the blood flow in an artery can be written as follows:

$$\left.\begin{aligned} &velocity\ is\ finite\ at\ r = 0, \\ &u'(r) = 0 \quad at \quad r = 0, \\ &No\ Slip\ velocity, u = 0 \quad at \quad r = h(z). \end{aligned}\right\} \quad (2)$$

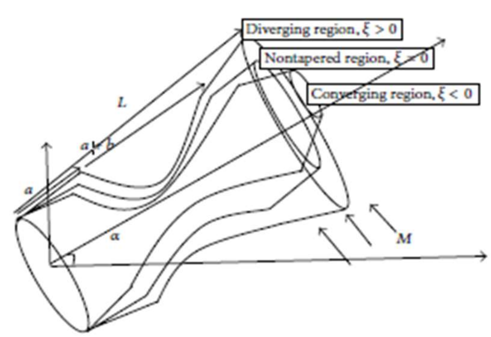

Figure2. Schematic of the tapered and stenosed artery.

Based on the assumptions made for the flow, equation (1) can be written as;

$$\rho g \sin\alpha = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) - \frac{\mu u}{\kappa} - \sigma {B_0}^2 u \quad (3)$$

where artery is inclined at an angle α, ρ is the density of fluid, inclined magnetic field is denoted by B_0.

Galerkin's Solution Method

Invoking the dimensionless variables, the dimensionless form of the equation can be written as:

$$\frac{Re}{Fr}\sin\alpha = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) - \left(\frac{1}{\kappa} + M^2\cos^2\theta\right)u \quad (4)$$

Setting pressure gradient,

$$-\frac{\partial p}{\partial z} = P(constant)$$

for the axisymmetric steady flow.

Since the order of differential equation is 2, so the polynomial considered should of degree three with four parameters. Let us assume the velocity u as:

$$u = c_1 + c_2 r + c_3 r^2 + c_4 r^3 \quad (5)$$

where c_1, c_2, c_3 and c_4 are the constants to be evaluated using boundary conditions. Using boundary condition we can write the equation (5) as follows:

$$u = c_3(r^2 - h^2) + c_4(r^3 - h^3) \quad (6)$$

After substituting equation (6), the residual function R can be written as follows;

$$R = \left[c_3(4 + \beta^2h^2 - \beta^2r^2) + c_4(9r + \beta^2h^3 - \beta^2r^3) - \left(\frac{Re}{Fr}\sin\alpha - P\right)\right] \quad (7)$$

where $\beta = \left(\frac{1}{\kappa} + M^2\cos^2\theta\right)$. Galerkin's method suggest that;

$$\int_0^h w_i\, R\, dr = 0,\ for\ i = 1,2, \quad (8)$$

where $w_1 = (-h^2 + r^2)$ and $w_2 = (-h^3 + r^3)$.

Substituting equation (7) in equation (8) with weights defined, equation (9) for $i = 1,2$ can be written as follows:

For $i = 1$;

$$\int_0^h (-h^2 + r^2)\left[c_3(4 + \beta^2h^2 - \beta^2r^2) + c_4(9r + \beta^2h^3 - \beta^2r^3) - \left(\frac{Re}{Fr}\sin\alpha - P\right)\right]dr = 0 \quad (9)$$

For $i = 2$;

$$\int_0^h (-h^3 + r^3)\left[c_3(4 + \beta^2h^2 - \beta^2r^2) + c_4(9r + \beta^2h^3 - \beta^2r^3) - \left(\frac{Re}{Fr}\sin\alpha - P\right)\right]dr = 0 \quad (10)$$

After solving equation (9) and (10), we can get the solution for constants c_3 and c_4 as follows:

$$c_3 = \frac{1}{12(40 + 8\,\beta^2h^2)}\left(\frac{Re}{Fr}\sin\alpha - P\right)\left[40 - 15(27h + 7\beta^2h^3)\left(\frac{-100800 - 2040\,\beta^2h^2}{(945h + 245\beta^2h^3)(540 + 105\beta^2h^2) - (378h + 90\beta^2h^3)(1440 + 288\beta^2h^2)}\right)\right]$$

$$c_4 = \left(\frac{-100800 - 2040\,\beta^2h^2}{(945h + 245\beta^2h^3)(540 + 105\beta^2h^2) - (378h + 90\beta^2h^3)(1440 + 288\beta^2h^2)}\right.$$

Hence the velocity expression becomes:

$$u = \frac{1}{12(40+8\,\beta^2h^2)}\left(\frac{Re}{Fr}\sin\alpha - P\right)\left[40 - 15\,(27h + 7\beta^2h^3)\left(\frac{-100800-2040\,\beta^2h^2}{(945h+245\beta^2h^3)(540+105\beta^2h^2)-(378h+90\beta^2h^3)(1440+288\beta^2h^2)}\right)\right](-h^2 + r^2) - \left(\frac{-100800-2040\,\beta^2h^2}{(945h+245\beta^2h^3)(540+105\ \ ^2h^2)-(378h+90\beta^2h^3)(1440+288\beta^2h^2)}\right)(-h^3 + r^3)\,, \quad (11)$$

where $\beta = \left(\frac{1}{\kappa} + M^2\cos^2\theta\right)$.

The volumetric flow rate can be now evaluated from:

$Q = 2\pi\int_0^h rudr.$

At r=h, the wall shear stress can be written as follows:

$\tau = \mu\frac{\partial u}{\partial r}.$

Result and Discussion

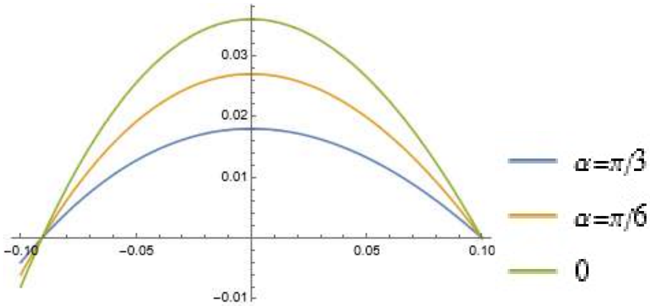

Figure 3. Axial velocity vs. angle of inclination of artery.

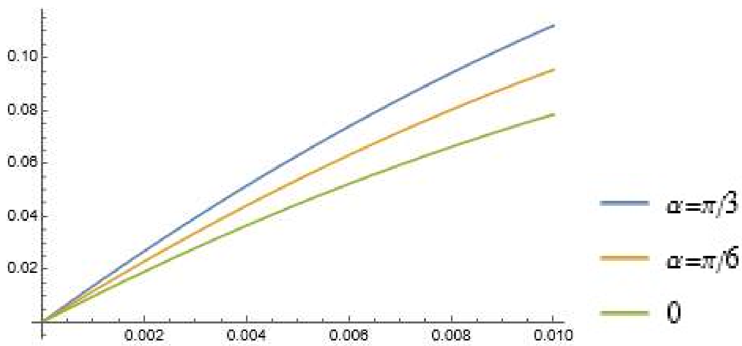

Figure 4. Shear stress in the blood flow with inclination.

Invoking Galerkin's method, we have analysed the influence of magnetic field on the flow pattern in stenosed tapered artery. Figure 3 represents that the flow velocity is less for the more inclined artery. This finding supports the result by Srivastava [1]. It can be visualised that the flow velocity is retarded by the inclination. Figure 4 represents the impact of inclination shear stress. This finding also is supported by Srivastava [1]. It can be visualised that the blood will experience more shear stress in the inclined artery.

Conclusion

In this study, Galerkin's method has been effectively used to investigate the influence of a magnetic field on blood flow through a stenosed, tapered, and inclined artery. The analysis reveals that arterial inclination plays a significant role in modifying flow behavior. Specifically, the results show that as the inclination of the artery increases, the axial velocity of blood flow decreases. This observation is consistent with the findings of Srivastava [1], affirming that inclination leads to a retardation in flow velocity.

Furthermore, the study demonstrates that the inclination of the artery contributes to an increase in shear stress along the arterial wall. The enhancement of wall shear stress in inclined arteries indicates a greater mechanical load on the vessel walls, which could have physiological implications in the progression of arterial diseases. These findings highlight the importance of accounting for arterial geometry and external magnetic influences in hemodynamic modeling and may assist in the development of more accurate diagnostic tools and treatment strategies for cardiovascular conditions.

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