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Illustration of Denton's Temporal Disaggregation Methods with the Application of Annual GDP of Bangladesh for Quarterly Benchmarking and therefore Forecasting

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ABSTRACT

Different methods of temporal Disaggregation are discussed in detail, mainly Denton's methods developed by Denton in 1971, and others are purely mathematical. In the beginning, Denton's original article describes the original method that Denton developed and its solution. Annual export (low-frequency series) of Bangladesh is disaggregated into quarterly series (high-frequency series) by the methods of Denton's additive and proportional (first and second difference), and Denton-Cholette additive and proportional (first and second difference) methods considering quarterly imports of capital goods & others as the indicator series for the fiscal year FY2009 to fiscal year FY2019. The R package "tempdisagg" [47] for temporal disaggregation of time series has been used for the temporal disaggregation of quarterly data. By comparing the estimated series with the real quarterly export with the aid of root-mean-squared errors (RMSEs), it comes to an end that the Denton-Cholette additive method (the first difference) performs better than the methods that are considered.

KEYWORDS

Denton, temporal disaggregation, indicator, additive, proportional.

1. INTRODUCTION

The researchers and policy makers often require data at shorter period or more frequently, though available yearly which are lower frequency data are usually more concise and detailed [19]. In practice, the quarterly or monthly time series data are not always available and the researchers face problem to derive these data by a proper method for further analysis. To disaggregate the yearly data into sub-annual series (quarterly or monthly), temporal disaggregation techniques are very useful tools. In this mechanism, quarterly or monthly (high frequency) time series data are derived from the yearly or quarterly (low frequency) time series data. An extensive review of temporal disaggregation of time series has been done by Pavia (2010) [45], and the comparison of different techniques was made as well.

France, Italy and some other European countries, computed quarterly data of Gross Domestic Product (GDP) by applying various disaggregation process as official statistics [1]. Besides, for econometric modelling, it could be suitable to disaggregate the lower frequency data by fitting a proper model at lower frequency without losing significance information [29]. Temporal disaggregation techniques are also very pragmatic in estimating and predicting economic data along with deriving high-frequency data [19]. For short-term analysis, temporal disaggregation methods are enormously used in variety of business and economic problems [43].*

When the disaggregated high frequency data is obtained with the help of one or more high-frequency indicator series, the procedure is known as Benchmarking [34]. Thus, Benchmarking is a specific case of temporal disaggregation method to disaggregate low frequency series into high-frequency series with the help of available indicator series.

Temporal disaggregation techniques are applied to interpolate or disaggregate the low frequency series to the higher frequency time series in such a way that the sum (for flow series), the average (for index series); or the first or the last value (for stocks series) of the resulting high-frequency series is consistent with the low frequency series [1]. For the flow series, the magnitude (size) of the observations are measured over a period of time such as GDP, export, import, national income, household income, expenditure, production, profit, savings, interest, household consumption, etc. are the example of flows data. On the other hand, stocks series measured the level of observations at a particular point in time such as unemployment, money stock, public sector debt, wealth, foreign debts, loan, inventories, opening stock, money supply, population, etc. It has to be mentioned that both stocks and flows can be expressed as the index series [19].

2. METHODS OF TEMPORAL DISAGGREGATION

To disaggregate the time series data, there are several methods are available [20]. The mathematical models and statistical models can be divided broadly into two categories.

2.1 The indicator (or related) high frequency series are not available:

When the available series are only the annual (high frequency) data, the related high frequency (sub-annual) series are not available, the methods are purely mathematical methods and more theoretically founded on the Autoregressive Integrated Moving Average (ARIMA) model [45]:

- *Mathematical and numerical smoothing methods*: The commonly used (mathematical and numerical) methods are smoothing methods are developed by Boot, Feibes, and Lisman (1967) [16] known as BFL method; and by Denton (1971) [26]. The cubic spline smoothing procedure is a frequently used numerical method [20].
- *Time series ARIMA models*: When indicator (related high frequency) series are not available, the ARIMA models are developed by Stram and Wei (1986) [48]; Wei and Stram (1990) [52]; and Barcellan and Di Fonzo (1994) [13] to generate sub-annual estimates of the time series data.

2.2 Both the low frequency series and the high frequency indicator (related) series are available:

When both the annual series (low frequency) and sub-annual series (high frequency-indicator series) are at hand, the suitable methods are:

- *Pure mathematical (numerical) and mathematical benchmarking methods (adjustment methods):* The pure mathematical methods are also known as adjusting methods and the most popular one is the Denton adjustment method by Denton (1971) [26]. The extensions of the Denton method is developed by Ginsburgh's (1973) [38]; Helfand, Monsour, and Trager (1977) [39]; Cholette (1979) [21]; Causey and Trager (1981) [18], Hillmer and Trabelsi (1987) [40], Bozik and Otto (1988) [17]. Trabelsi and Hillmer (1990) [51], Cholette and Dagum (1994) [23], Di Fonzo (2002) [28]; and Di Fonzo and Marini (2005) [30].
- *Statistical methods (Optimal methods):* This approach was first applying by Friedman (1962) [37] and Chow and Lin (1971) [24] by extending the Friedman’s result developed the Chow-Lin regression methods [45]. Chow-Lin regression methods are the most widely used statistical methods which were further expanded by Fernandez (1981) [35]; and Litterman (1983) [44]. Furthermore, to improve accuracy of estimates of Chow-Lin several authors (e.g., Salazar et al. 1997 [46], Weale 1997 [53]; and Silva, Santos and Cardoso 2001 [49] have proposed to generalize Chow-Lin approach (including Fernández and Litterman extensions) by the use of linear dynamic models, explained by Pavia (2010) [45]. Di Fonzo (1990) [27] worked on Chow-Lin to generalize for multivariate extensions for more than one series. The others model-based statistical models are the generalized regression-based methods (Dagum and Cholette 2006) [25]; time-series ARIMA models (Hillmer and Trabelsi 1987) [40] and state space models [32]) cited by IMF (2017) [42]. Besides, Al-Osh (1989) [2], Wei and Stram (1990) [52] linked an ARIMA model-based technique in a regression model to surpass some arbitrariness in the selection of the stochastic structure of the high frequency disturbances with the use of sub-annual or related series, stated by Di Fonzo (2003) [29].

The basic methods for temporal disaggregation are Denton (1971) [26], Dagum-Cholette (1994) [25]; and Chow-Lin (1971) [24] and its variants Fernandez (1981) [36] and Litterman (1983) [44]. Among these, Denton and Dagum-Cholette and its variants (Dagum and Cholette, 2006 [25]) are basically deal with movement preservation. These methods producing the disaggregated series which follow the trend of the given indicator, not considering the relationship with the low frequency series.

In this paper, the high frequency series (quarterly series) are used as the indicator (or related) series to generate the disaggregated desired high frequency time series data, along with the low frequency annual time series. The Denton adjustment method and its variant along with the Dagum-Cholette method will be explained and applied to analyse the data. Finally, the two methods will be compared for the accuracy of disaggregation ability. It is also mentioned that in the data analysis, our concern will be only about flows variables (e.g., GDP, export, income, consumption etc.), where low frequency annual time series are simply aggregates of sub-annual quarterly (high frequency) series.

3. TEMPORALLY DISAGGREGATION WITH HIGH FREQUENCY INDICATOR (RELATED) SERIES: DENTON ADJUSTMENT METHOD AND ITS VARIANTS

Denton's (1971) method is the one of the most (not only among adjusting procedures) predominant methods in the area of temporally disaggregation [20]. Denton (1971) method initially estimate the desired series by minimizing a squared loss function. The crucial part in Denton's proposal is the selection of the symmetrical matrix which determining the specific forms of the squared loss function. Denton concentrated on the solutions obtained by minimizing the h^{th} differences between the series which to-be estimated and the initial approximation and found by [16] as a particular case of his algorithm. Later on, [22] proposed a slight modification to the squared loss function of Denton' method which is not dependent on the initial conditions. Although, nevertheless, the basic extensions of Denton approach were done by [40], [51], [23], [28] and [30] and they extended the method to adoptable algorithm and extended it to the multivariate case [45].

Adjustment method: Denton's suggested methods are the least-squares-based models, which preserve the movement of the indicator series [41]. Denton's (1971) [26] proposed the procedure to deal with the adjustment problem with given constrain and recommended different techniques to avert the step problem. Let

x_t = annual low frequency (LF) series of T years to be disaggregated for t = 1, 2, ….., T

$x = (x_1, x_2, \ldots\ldots, x_T)'$

Where x is a $(T \times 1)$ column-vector.

Let us assume that the annual series x_t will be disaggregated into k periods for each year, and k being an integer. As the annual series (low frequency) will be disaggregated into quarters, then the periods per year will be 4, that is k=4. The estimated as sub-annual quarterly series as well as the given indicator sub-annual quarterly series consist of total $n = kT = 4T$ values for the total T years.

Again, let us consider that z_i is the known quarterly indicator or related sub-annual high frequency (HF) series, for i=1, 2,….n; and can be represented as $(kT \times 1) = (4T \times 1) = (n \times 1)$ column-vector.

$$z = (z_1, z_2, \ldots\ldots, z_n)'$$

And y_i be the desired sub-annual quarterly high frequency (HF) series to be estimated; for i= 1, 2,…n; where $n = 4T$; and also can be represented as $(4T \times 1) = (n \times 1)$ column-vector.

$$y = (y_1, y_2, \ldots\ldots, y_n)'$$

Denton method follows two core rules: (i) to find the estimates y, which minimize of distortion in the indicator (or preliminary/original) series z, and (ii) in a specific year, the sum of the quarterly (as k=4) values of the estimated (disaggregated) new series of y will be equal to that yearly total. More generally, the original Denton adjustment method states that the final estimates of sub-annual quarterly (high frequency) series y, by minimizing the penalty function p(y, z), with the restrictions (1):

$$x_t = \sum_{i=4t-3}^{4t} c_{ti} y_i \qquad for\ t = 1,2 \ldots.\ ,T \tag{1}$$

Where

y_i = the unknown quarterly values to be estimated; and

x_t = given yearly total in year *t*; and

c_{ti} denotes the weights of coverage fraction of sub-annual estimate of quarterly (or monthly) series for year *t* and *ith* quarter; and for the disaggregation of yearly to quarterly series (k=4) is the $(k \times 1) = (4 \times 1)$ vector has the following form [33]:

Flow series: $c = (1, 1, 1, 1)'$

Index series: $c = (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})'$

Stock, BOP (Beginning of Period) series: $c = (1, 0, 0, 0)'$

Stock, EOP (End of Period) series: $c = (0, 0, 0, 1)'$

As the analysis of this paper will be carried out for the flows variables (e.g., GDP, export, income, consumption etc.), where low frequency annual time series are simply aggregates of sub-annual quarterly (high frequency) series, thus $c_{ti} = 1$; and the temporal disaggregation constraints (1) can be written as

$$x_t = \sum_{i=4t-}^{4t} y_i \qquad for\ t = 1, 2 \ldots.\ ,T \tag{2}$$

To generalize, the temporal disaggregation constraints (1) in matrix form as notation as :

$$x = Cy \tag{3}$$

Where C is the $(T \times n)$ temporal aggregation matrix that links the given (observed) annual series of $(T \times 1)$ vector *x* to the corresponding estimated (unknown) sub-annual series of $(n \times 1)$ vector. Basically, aggregation matrix *C* converting the high-frequency *y* in low frequency data *x* with k temporal aggregation orders [30], has the following structure:

$$\boldsymbol{C} = \begin{bmatrix} c_1 & c_2 & \cdots & c_k & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & c_1 & c_2 & \cdots & c_k & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & c_1 & c_2 & \cdots & c_k \end{bmatrix}$$

$$\boldsymbol{C} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} \otimes (c_1, c_2, \ldots\ldots, c_k) = I_T \otimes c' \qquad (4)$$

where I_T is an identity matrix of order $(T \times T)$ and $c = (c_1,\ c_2,\ \ldots\ldots,\ c_k)'$ is a column vector of $(k \times 1)$ and $\otimes$ denote the Kronecker product. As the disaggregated quarterly series will be added to the annual series; and for the flow series of order=4, the temporal aggregation matrix *C* can be written as:

$$C = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} \otimes [1\ \ 1\ \ 1\ \ 1] = I_T \otimes c_4' \qquad (4a)$$

where $c_4 = (1,\ 1,\ 1,\ 1)'$ is a (4×1)column vector of unit and I_T an identity matrix of $(T \times T)$. The aggregation matrix *C* can be explained as if the columns indicate the quarterly value in a given year is a 1, otherwise a zero, where the years are indicated by the rows [16]. In case of other series such as index and stock series, the temporal aggregation matrix *C* of order $k = 4$, has the following forms:

Index series: $$C = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} \otimes [\tfrac{1}{4}\ \ \tfrac{1}{4}\ \ \tfrac{1}{4}\ \ \tfrac{1}{4}] = I_T \otimes c_4'$$

Stock, BOP (Beginning of Period) series:

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 1 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} \otimes [1\ \ 0\ \ 0\ \ 0] = I_T \otimes c_4'$$

Stock, EOP (End of Period) series:

$$C = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} \otimes [0\ \ 0\ \ 0\ \ 1] = I_T \otimes c_4'$$

It is noted that $n = kT = 4T$, but if $n > kT$ an extrapolation issue has to be considered, the difference $(n\text{-}kT)$ being the number of high frequency sub-annual quarterly estimated series for which the series is not subject to temporal disaggregation constraints $x = Cy$; and an additional $(n - kT)$ columns of zeroes are added to the matrix *C* [33]:

$$C = \begin{bmatrix} \cdots & 0 & 0 & \cdots & 0 \\ \cdots & 0 & 0 & \cdots & 0 \\ \ddots & \vdots & \vdots & \ddots & \vdots \\ \cdots & 0 & 0 & \cdots & 0 \end{bmatrix} \qquad (4A)$$

The first part of the aggregation matrix *C* depends on the nature of the series (such as flow, index or stock series).

Estimation of the Disaggregated (sub-annual) series: Let *M* be a positive define symmetric $(n \times n)$ non-singular weighting matrix (later will be defined), then the objective function of original Denton model is specified as:

$$Min_y P(y, z) = (y - z)'M(y - z) \qquad (5)$$

where $(y - z)'M(y - z)$ is a quadratic form in the differences between the indicator (preliminary) and the estimated (disaggregated) sub-annual time series, which (5) must be optimized by fulfilling the constraints (3).

$$x = Cy$$

The objective function (5) can be written as a Lagrangian expression as:

$$f(y, \lambda) = (y - z)'M(y - z) - 2\lambda'(x - Cy) = y'My - 2y'Mz + z'Mz - 2\lambda'x + 2\lambda'Cy \qquad (6)$$

where $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_T)'$ is the $(T \times 1)$ column vector, which contains Lagrange multipliers associated with the linear constraints $(x - Cy)$. By taking partial derivatives of the objective function (6) with respect to y and λ, and equating to zero, the normal equations are

$$\frac{\delta(f(y,\lambda))}{\delta y} = 2My - 2Mz + 2C'\lambda = 0$$

$$\frac{\delta(f(y,\lambda))}{\delta \lambda} = 2Cy - 2x = 0$$

The above equations can be written as [25]:

$$My + C'\lambda = Mz$$

$$Cy = x = Cz + (x - Cz)$$

The above equations again can be expressed as [25]:

$$\begin{bmatrix} M & C' \\ C & 0 \end{bmatrix} \begin{bmatrix} y \\ \lambda \end{bmatrix} = \begin{bmatrix} M & 0 \\ C & I \end{bmatrix} \begin{bmatrix} z \\ x - Cz \end{bmatrix}$$

Where I is the identity matrix of $(T \times T)$ and 0 is null matrix of $(T \times T)$. This a system of $(n + T)$ linear equations (augmented) of the general form in $(n + T)$ unknowns of y_n and λ_t [16] is obtained for the penalty-minimizing solution [26] as:

$$\begin{bmatrix} y \\ \lambda \end{bmatrix} = \begin{bmatrix} M & C' \\ C & 0 \end{bmatrix}^{-1} \begin{bmatrix} M & 0 \\ C & I \end{bmatrix} \begin{bmatrix} z \\ x - Cz \end{bmatrix} \qquad (7)$$

Then the solution for y will be

$$y = z + M^{-1}C'(CM^{-1}C')^{-1}(x - Cz) \qquad (8)$$

Or $\quad y = z + A(x - cz)$

where $A = M^{-1}C'(CM^{-1}C')^{-1}$

Thus, the disaggregated series are the sum of the given indicator (also known as preliminary series) series and linear combinations of the differences between the yearly value of given annul series and yearly total of the indicator series [26]. In other words the preliminary estimates are adjusted by distributing the aggregated discrepancies according to the matrix of weights A.

If M is an identity matrix of $(n \times n)$, the minimization is done from the sum of squares of simple difference between the original (preliminary or indicator) and the revised (estimated) values. Thus,

$$A = M^{-1}C'(CM^{-1}C')^{-1} = I^{-1}C'(CI^{-1}C')^{-1} = C'(CC')^{-1} = (1/k)C' = (1/4)\ C',$$

For disaggregation of yearly series to quarterly series , which means the difference for each period (year) is allocated identically over the 4 quarters with the same amounts and does not give a satisfactory result for the estimated series y; and therefore different variants are proposed by Denton (1971) [26].

4. VARIANTS OF DENTON METHOD

Denton proposed several alternatives of the original model. The variants are based on the first or higher order difference of the estimated sub-annual series and the indicator series. The additive first differences (AFD) and proportional first differences (PFD); and also the additive second differences (ASD) are the most widely used variants [20]. The additive method preserves the period-on-period differences and the proportional (or multiplicative) method preserves the period-on-period growth rates. The variants are shortly described below:

- ***Additive first difference (AFD):*** For this variant, the additive correction (the first difference) $\Delta(y_i - z_i)$, that is the difference between the final estimates and the indicator series, $(y_i - z_i)$, $i = 1,2 \ldots \ldots, n$, is as constant as possible under the temporal aggregation constraint (1). As a result, the estimated sub-annual series, y_i tends to be similar to the given sub-annual related series z_i. The penalty function (or objective function) is

$$p(y,z) = \sum_{i=1}^{n}[\Delta(y_i - z_i)]^2$$

- ***Proportional first difference (PFD):*** When the proportional changes between the estimated and the indicator series $\Delta(y_i/z_i)$ are constant under the temporal aggregation constraint (1), the variant of the Denton's original model is known as Denton's proportional first difference (PFD). Under proportional first difference variant, the final sub-annual estimates y_i will have same period-to-period growth rate as the given indicator series z_i and the penalty function is as follows:

$$p(y,z) = \sum_{i=1}^{n}[\Delta(y_i/z_i)]^2$$

$$or \qquad p(y,z) = \sum_{i=1}^{n} \left(\frac{y_i}{z_i} - \frac{y_{i-1}}{z_{i-1}}\right)^2$$

- ***Additive second difference (ASD):*** The objective function of the additive second difference (ASD) is obtained by minimizing sum of squares of the second order difference $\Delta^2(y_i - z_i)$ along with the linearity in the additive correction $\Delta(y_i - z_i)$ as much as possible as following:

$$p(y,z) = \sum_{i=1}^{n}[\Delta^2(y_i - z_i)]^2$$

- ***Proportional second difference (PSD):*** For the case of the proportional second difference (PSD) variant, the penalty function of the second order difference $\Delta^2(y_i/z_i)$ is minimized by retaining changes in the ratio $\Delta(y_i/z_i)$ as linear as possible as follows:

$$p(y,z) = \sum_{i=1}^{n} \left[\Delta^2 \left(\frac{y_i}{z_i}\right)\right]^2 = \sum_{i=1}^{n} \left[\Delta \left(\frac{y_i}{z_i} - \frac{y_{i-1}}{z_{i-1}}\right)\right]^2$$

Explanation and solution of Denton Additive First Difference (AFD) and Proportional First Difference (PFD); other variation of Denton' PFD; and the enhancement of Proportional Denton Method for extrapolation is attached as a supplement which contains equation (9) to (28).

5. THE CHOLETTE–DAGUM REGRESSION-BASED METHOD (OR DENTON-CHOLETTE METHOD)

The crucial challenge for the researches to improve the efficiency of the estimates by reducing estimation errors and they face basically three big problems as: (i) auto correlated problems occurred as the time series data are collected from repeated surveys by total or partial projecting the sample, (ii) heteroscedasticity errors aroused because of the surveys are usually are designed to keep the coefficient of variations (or the precision) of the estimates as steady as possible over time due to dissimilarity in variation or variance; and finally (iii) the presence of biased due to non-response, breakdown of frame deterioration over time, autocorrelation, etc. [23]. Taking into account the above three problems, Cholette and Dagum (1994) [23] developed a generalized least-square regression model of benchmarking method and thus a very adaptable model in the field of temporal disaggregation [42]. The benchmarking (Additive) model which is suggested by Cholette and Dagum (1994) [23] is bases on the two equations (29a) and (29b) as follows:

$$z_i = a_i + y_i + e_i \quad \text{for } i = 1,2,\ldots,n \qquad (29a)$$

$$x_t = \sum_{i=4t-}^{4t} y_i + w_t \qquad for\ t = 1,2,\ldots,T \qquad (29b)$$

Given that

$E(e_i) = 0, E(e_i e_{i-h}) \neq 0$
$E(w_t) = 0, E(w_t w_{t-h}) \neq 0, E(w_t^2) = \delta_t^2$; and

$E(e_i w_t) = 0$.

Where

$n = 4T$, T is the total years in yearly data series and thus, n is total number of quarterly series

z_i is the available quarterly indicator or related high frequency (HF -sub-annual) series;

y_i is the real quarterly values to be estimated but unknown;

a_i is a combined deterministic effect which is an unknown constant bias parameter to be estimated;

e_i is the quarterly auto correlated and heteroscedastic term related with the given indicator series and is expected to have zero mean and a known auto covariance structure;

x_t is the given yearly value in year t of T years (the observed benchmarks); and

w_t is the annual heteroscedastic error in the given annual series x_t, and is independent of the error e_i.

The Cholette–Dagum regression-based depends on the assumptions for a_i , e_i and w_t . It is necessary that the annual heteroscedastic error w_t to clarify the status whether the given yearly series (benchmark series) is liable to error as well. The annual series x_t will be nonbinding as they are subject to adjust (error) in the benchmarking procedure (or temporal disaggregation); and the annual series x_t will be binding when they are not subject to adjust (error) and thus $E(w_t^2) = 0$ [23]. The annual series are usually obligatory constraints for the quarterly series and thus binding [42].

The deterministic effect a_i is generally computed from a set of deterministic explanatory variables (regressors) denoted by $\boldsymbol{r_{i,h}}$ as follows: multiplied by their corresponding that is,

$$a_i = \sum_{h=1}^{k} r_{i,h}\beta_h \qquad for\ i = 1, 2, \ldots \ldots \ldots n$$

where β_h are regression coefficients , k is the number of deterministic effects under consideration. The deterministic regressors $\mathbf{r_{i,h}}$ usually comprise of a constant k=1 and $r_{i,h} = -1$ to brace the average level discrepancy between the yearly and quarterly data [23].

The role of the error term e_i is very vital and is calculated from the quarterly discrepancy between the estimated series y_i and the given quarterly series z_i. The error e_i is also known as survey error [23]. The error term e_i has two properties: (i) It is proportional to the indicator series z_i, which ensures to allot the errors evenly throughout the indicator; and (ii) It will exhibit smooth movements from one quarter to another to make the movement of y_i and z_i are very adjacent to each other. These two feature of e_i are required to fulfill the key objective of benchmarking or disaggregation [42]. The error e_i is standardized to e'_i by the given quarterly series z_i in order to attain a proportional adjustment as follows:

$$e'_i = \frac{e_i}{z_i}$$

As it is guessed that the standard deviation of e_i is same as the indicator z_i and this tends to the constant coefficient of variation $\sigma_i/z_i = 1$ for any quarter i, where σ_i is the standard deviation in quarter i [42]. The standardized error e'_i exhibits the autoregressive model of order one or AR (1):

$$e'_i = \varphi e'_{i-1} + \upsilon_i \qquad (30)$$

Where φ is the autoregressive parameter with $|\varphi| < 1$, and the υ_i's are transitions and identically and independently distributed (i.i.d.):

$E(\upsilon_i) = 0, E(\upsilon_i^2) = 1, E(\upsilon_i\upsilon_{i-h}) = 0$ for any i and h

Now the system of equations (29a) and (29b) be can be written as the following matrix form [23]:

$$\begin{bmatrix} \mathbf{z} \\ \mathbf{x} \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{C} \end{bmatrix} \begin{bmatrix} \mathbf{a} \\ \mathbf{y} \end{bmatrix} + \begin{bmatrix} \mathbf{e} \\ \mathbf{w} \end{bmatrix}$$

Where

1 is a $(n \times 1)$ vector of ones,

C is the $(T \times n)$ temporal aggregation or design matrix as defined in equation (4a) for the quarterly flow series;

z, x, and y are the column vectors of order $(n \times 1)$, $(T \times 1)$ and $(n \times 1)$ respectively as earlier defined in the Denton's method; and

a, e, and w are the vectors of order $(n \times 1)$, $(n \times 1)$ and $(T \times 1)$ respectively

It has to be mentioned that Cholette and Dagum (1994) [23] assumed the variance–covariance matrix of the quarterly error w_t to zero in order to obtain the solution for the disaggregated series. Then the solution for the disaggregated quarterly series y by to the Cholette–Dagum proportional benchmarking with AR error [42] is

$$y = z^* + VC'(CVC')^{-1}(x - Cz^*) \qquad (31)$$

Where

z^* is the (n×1) vector with the bias-adjusted indicator series z_i^*, which is calculated in equation (29a),

$V = z^*(\Omega^{-1})z^*$ is the (n×n) variance–covariance matrix of the quarterly error e_i ,

z^* is the (n×n) diagonal matrix, which contains the values of the bias-adjusted indicator series z^*_i in the main diagonal; and

$\Omega = \omega'\omega$ is the autocorrelation matrix of the AR(1) model with parameter ϕ , where

$$\omega = \begin{bmatrix} \sqrt{1-\varphi^2} & 0 & 0 & \cdots & 0 \\ -\varphi & 1 & 0 & \cdots & 0 \\ 0 & -\varphi & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

Where, φ is the autoregressive parameter with $|\varphi| < 1$ is defined in (30).

It has to be mentioned that the Cholette–Dagum method with AR error minimizes an objective function which is similar to the proportional Denton method first difference (equation (17)) is as shown below [42]:

$$min_{y_i}\left[\left(\frac{1}{1-\varphi^2}\right)\left(\frac{y_1}{z_1^*}\right)^2+\sum_{i=2}^{n}\left(\frac{y_i}{z_i^*}-\varphi\frac{y_{i-1}}{z_{i-1}^*}\right)^2\right] \quad (32)$$

Where z_i^*, y_i and z_i are defined as before. From the equation (32), it is clearly visible that the AR parameter φ performs a very import act in retaining the short-term movements of the related or indicator series in addition to forecast (extrapolation). For keeping the movement preservation as in the Denton technique, the value of φ is preferred to be in a range between 0.71 and 0.93 [42].

Cholette and Dagum (1994) [23] stated that the bias ***a*** is deterministic, If the above regression method is used for all over series, otherwise the bias ***a*** is stochastic, if the method is applied on moving estimation intervals (of 5 years say), because it advances according to the changes when entering each interval.

6. RESULTS AND ANALYSES: DISAGGREGATED THE EXPORT OF BANGLADESH AS A REAL WORLD EXAMPLE

Bangladesh Bank, which is the central bank of Bangladesh along with Bangladesh Export Promotion Bureau updated the export data every month, and then compiled it to quarterly and yearly basis. The estimation of disaggregated quarterly series is carried out using the entire data span of yearly export from the fiscal year 2009 to second quarter of fiscal year 2019 by using Denton's additive and proportional (first and second difference), and Denton-Cholette additive and proportional (first and second difference) methods aiding with the indicator series.

It has to be mentioned that actual quarterly export of Bangladesh is already available in the estimation period and thus will be helpful to compare the resulted disaggregated series. Data of two quarters of the fiscal year 2019 are chosen to extrapolate the export for two quarters by the above temporal disaggregation methods. The study will assess the disaggregation quality and as well as forecasting ability of two methods from different points of view. It has to be acknowledged that fiscal year In Bangladesh is started from 1 July and ended on 30 June of the next year. Thus the estimation period of fiscal 2009 to second quarter of fiscal year 2019 will actually be covered data form 1 July 2008 to 31 December 2018.

Quarterly imports of capital goods & others (In million USD) will be treated as the indicator series. The reason that the import of these items are mainly used for production purposes and the surplus production is exported after mitigating domestic demand and thus closely related with the export. Imports of capital goods & others included the items: i. Capital machinery, iii. Others capital goods, and iii. Iron, steel & other base metals. It is noted that iron, steel & other base metals were under the category of capital goods & others up to the fiscal year 2015 but form the fiscal year 2016 steel & other base metals are included to the category of consumer & intermediate goods. To keep the consistency of the data, steel & other base metals are also added with capital goods & others form the fiscal year 2016 and onward in this analysis.

For the temporal disaggregation of quarterly data the R package "tempdisagg" [47] for temporal disaggregation of time series has been used. The other popular package for temporal disaggregation is ECOTRIM. The ECOTRIM is developed by the European Commission [14] and it supplies a set of mathematical and statistical techniques to carry out the temporal disaggregation of time series. The ECOTRIM deriving disaggregated time series, when aggregated data (that is lower-frequency data are available) as well as indicator series is available. By ECOTRIM still now, it is not possible to use Denton's method to disaggregate the time series data (Sax and Steiner 2013), although this package can easily be used to disaggregate time series data for other methods (e.g., Chow-Lin, Fernandez). The R package "tempdisagg" has overcome this limitation. The disaggregation results are discussed in details as follows:

In this paper, firstly yearly export will be disaggregated to quarterly export by Denton's and Denton-Cholette temporal disaggregation methods with indicator series and then resulted disaggregated series will be compared with the actual series. For further analysis of the disaggregation results, the quarterly (lag of one quarter) and annual (lag of four quarters) growth rates which are known as T-1 and T-4 growth rates are calculated for the resulted series and for the actual export and indicator series as well. Finally, by calculating the root-mean-squared errors (RMSE) for the growth rates for different models, the best model will be suggested for disaggregation and in addition to forecasting for short term periods.

7. DENTON'S ADDITIVE VARIANTS:

The disaggregation results by Denton's additive first (Denton.additive1) and second difference (Denton.additive2) are compared with the original quarterly series of export (export.q) in Fig below. And the T-1 and T-4 growth rates of the disaggregated quarterly export are correlated with the T-1 and T-4 growth rates of the original quarterly export are shown as well as with the indicator series in Fig 2 and Fig 3 respectively.

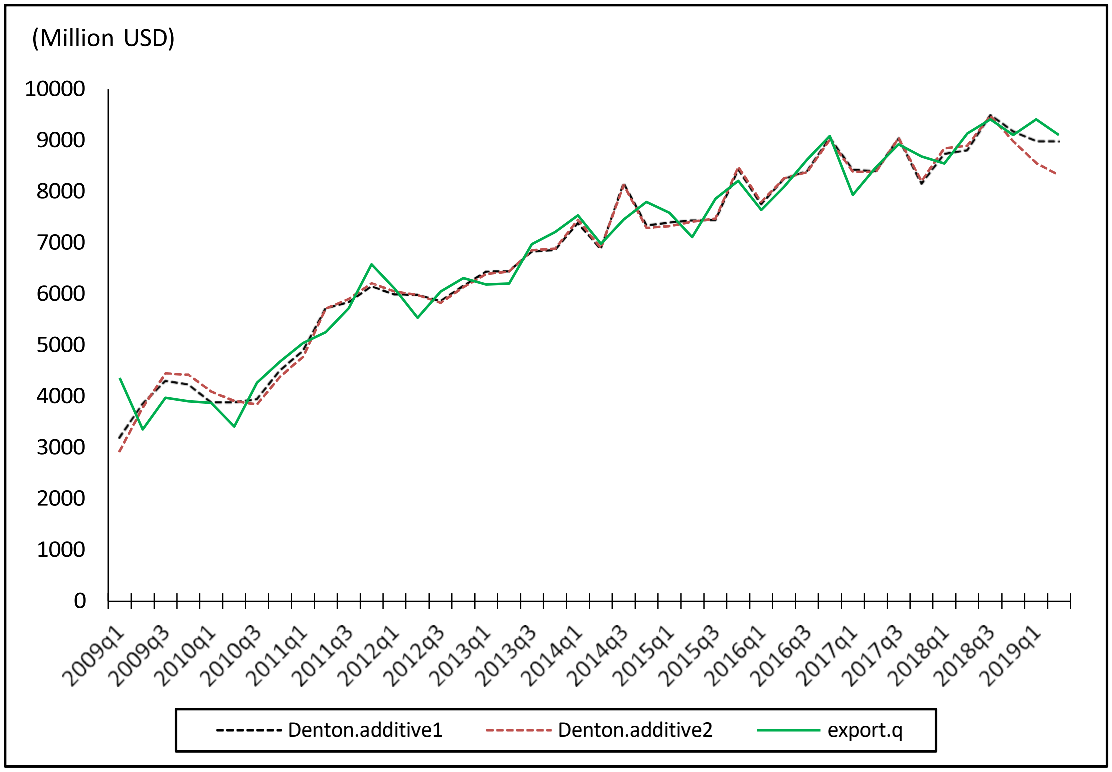

Fig 1. Disaggregated quarterly export by Denton's additive first and second difference, and actual quarterly exports from 2009q1 to 2019q2

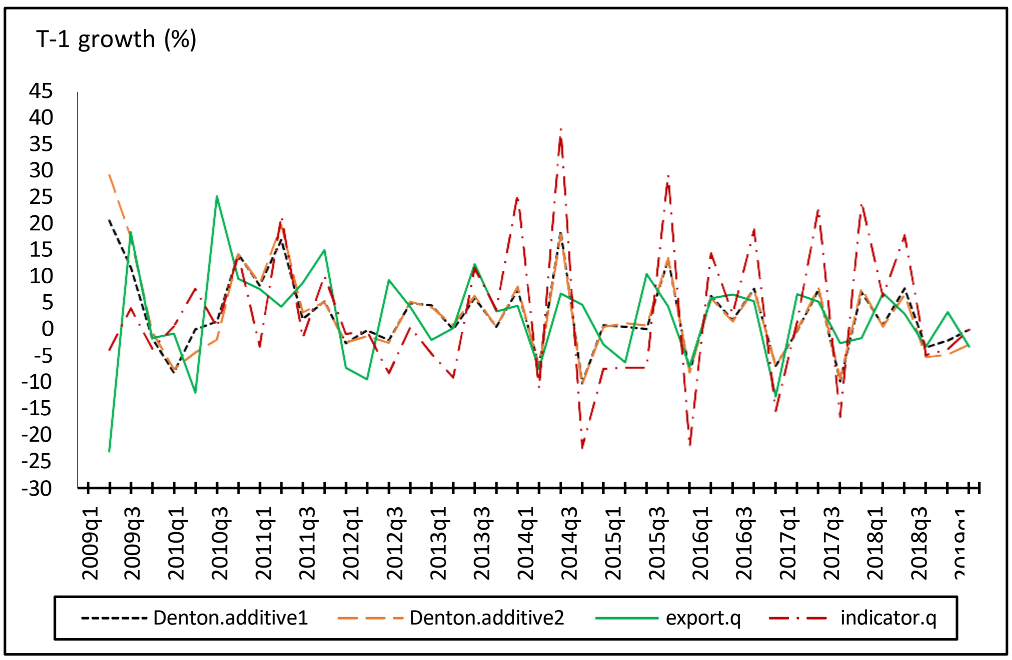

Fig 2. T-1 growth rates (quarterly) of disaggregated quarterly exports by Denton's additive first and additive second difference, actual quarterly export and indicator series 2009q1 to 2019q2

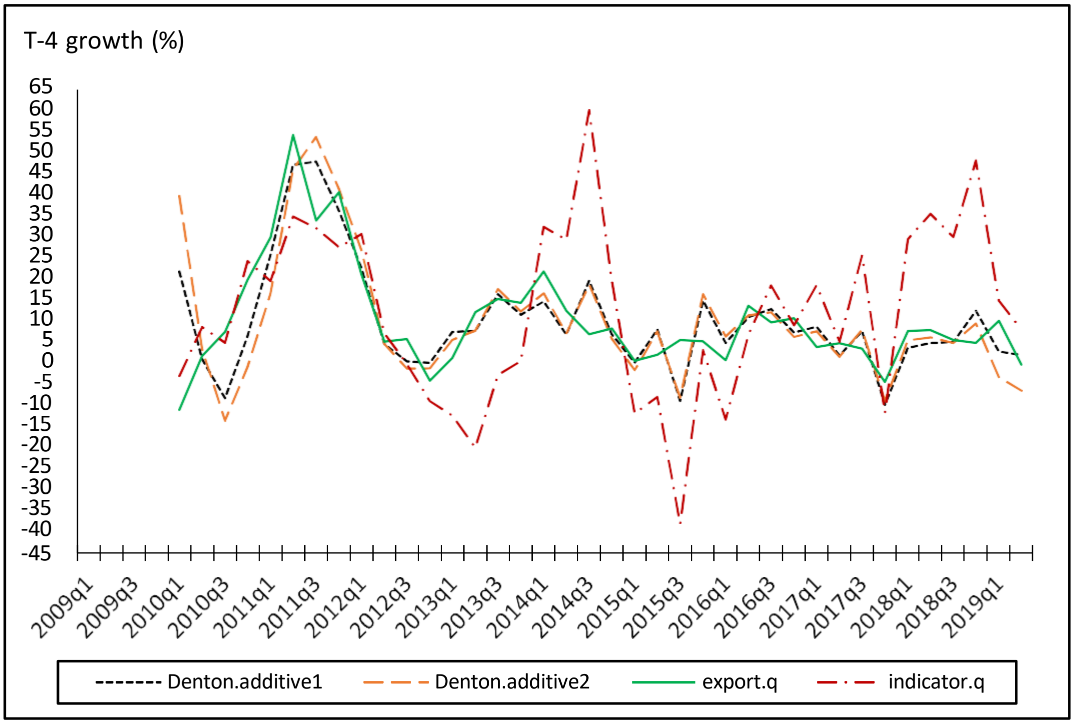

Fig 3. T-4 growth rates (annual) of disaggregated quarterly exports by Denton's additive first and additive second difference, actual quarterly export and indicator series from 2009q1 to 2019q2

Fig 1 represents the disaggregated exports by Denton's additive first and Denton's additive second difference are very close to each other and not so much far from the original quarterly export all the ways except at the beginning and at the end of the series. From the Figure 2 and Figure 3, it is evident that both of the T-1 and T-4 growth rates of the indicator series are well followed by the Denton's additive first and Denton's additive second although for T-1 growth rates of indicator series are much closer than the T-4 growth rates. On the other hand, the T-4 growth rates of the original quarterly export is much closer and similar pattern to the disaggregated series. The accuracy of the disaggregated series by Denton's additive first and Denton's additive second difference in terms of the original series as well as indicator series will be clearer from the comparison of the root-mean-squared errors (RMSE) as discussed in the followings.

At a glance the results of the RMSEs between the Denton's additive variants and the original quarterly export as well as with T-1 and T-4 growth rates of quarterly export and the indicator series are displayed in the Table 1.

Table 1. RMSEs between the Denton's additive variants and the quarterly export as well as with indicator series

		RMSE between	
	Series	**Quarterly export by Denton's additive 1st difference (Denton.additive1)**	**Quarterly export by Denton's additive 2nd difference (Denton.additive2)**
Original (millionUSD)	Quarterly export (export.q)	357.85	424.46
T-1 growth (lag one)%	Quarterly export (export.q)	10.37	11.43
T-4 growth (lag fours)%	Quarterly export (export.q)	4.43	11.79
T-1growth (lag one)%	Quarterly indicator (indicator.q)	9.67	10.56

T-4 growth (lag fours)%	Quarterly indicator (indicator.q)	17.90	19.64

For the case of absolute values (in million USD), Denton's additive first difference variant produced more accurate results than the Denton's additive second difference as the RMSE between the resulted quarterly export by the Denton's additive first difference and the actual quarterly export (357.85) is smaller than the counterpart RMSE between the resulted quarterly export by the Denton's additive second difference and the actual quarterly export (424.46). In terms of the growth rates(T-1 and T-4), Denton's additive first difference variant also outplays the Denton's additive second difference, though RMSE for the T-1 growth rates are little bit closer but for the T-4 growth rates, Denton's additive first difference produced much more perfect results than the Denton's additive second difference. In addition, it is also revealed (from table 1) that the Denton's additive first difference is also perform better when the RMSEs of the growth rates are made in term of the indicator series. Thus, we can conclude that in comparison to the additive first and additive second difference, Denton's additive first difference generated closer series than the second difference.

7.1 Denton's proportional variants:

The comparison between the original quarterly series of export (export.q) and the resulted quarterly exports obtained by Denton's proportional first difference (Denton.prop1) and second difference (Denton.prop2) in Fig 4 is revealed that the disaggregated series are very closer to each other except at the end of the series and are not following the original quarterly export very smoothly.

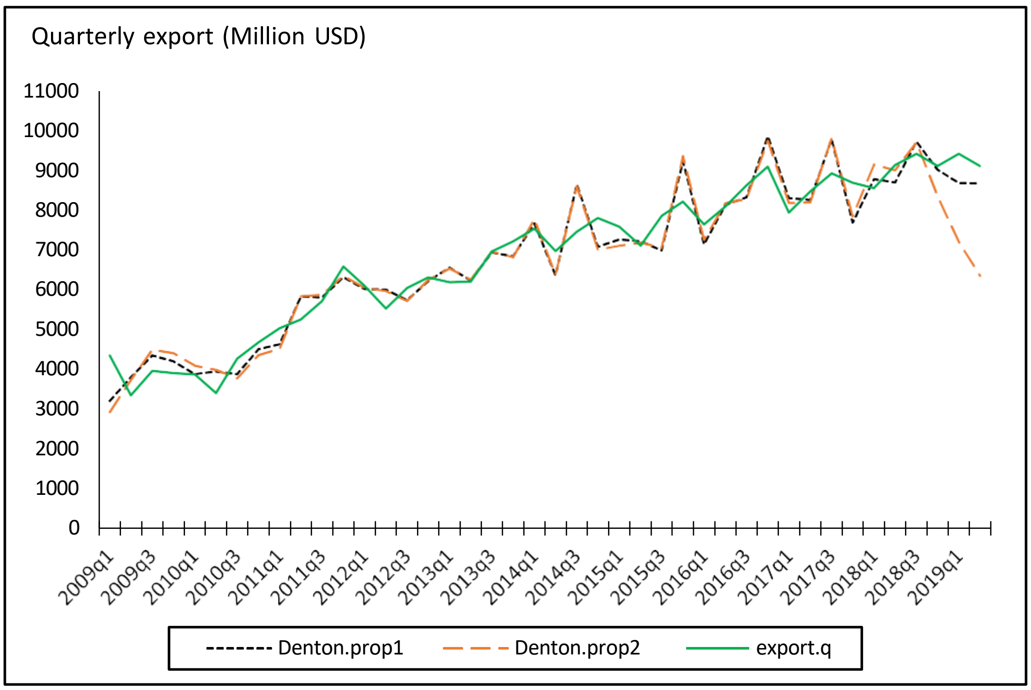

Fig 4. Disaggregated quarterly export by Denton's proportional first and proportional second difference, and actual quarterly exports from 2009q1 to 2019q2

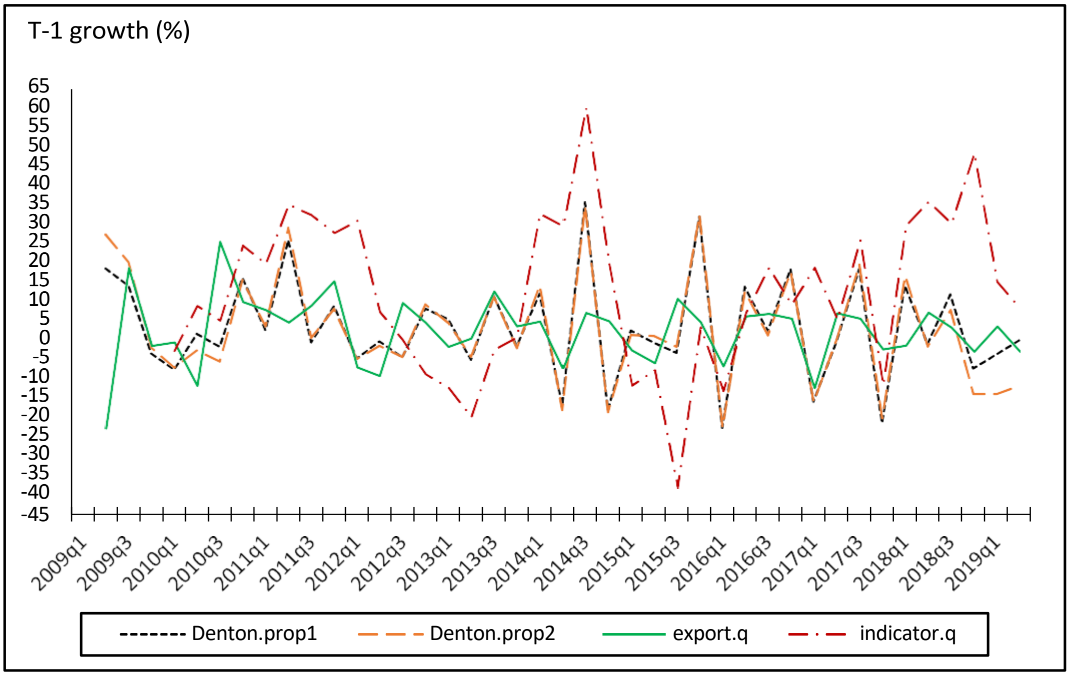

Fig 5. T-1 growth rates (quarterly) of disaggregated quarterly exports by Denton's proportional first and proportional second difference, actual quarterly export, and indicator series from 2009q1 to 2019q2

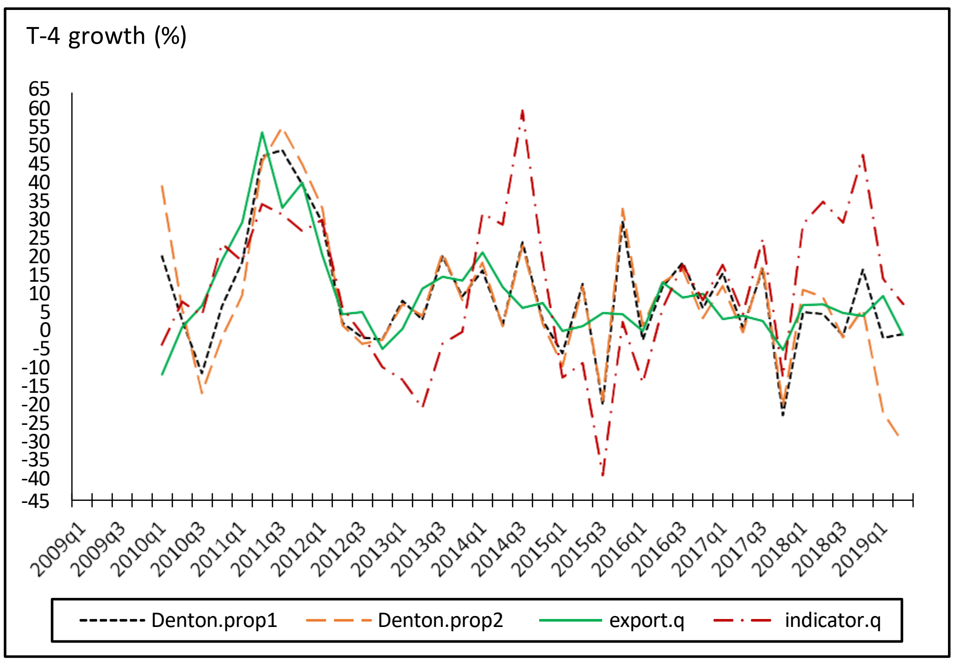

Fig 6. T-4 growth rates (annual) of disaggregated quarterly exports by Denton's proportional first and proportional second difference, actual quarterly export, and indicator series from 2009q1 to 2019q2

In terms of the T-1 and T-4 growth rates (Fig 5 and Fig 6), it is visualized that the T-1 growth rates both for the Denton's proportional first difference and second difference illustrates original quarterly export along with the indicator series better than the T-4 growth rates of the disaggregated quarterly export. The performance of the two variants of the Denton's proportional method correspond to the T-1 and T-4 growth rates will be concretely lucid from the root-mean-squared errors (RMSE) which are displayed in Table 2.

Table 2. RMSEs between the Denton's proportional variants and the quarterly export as well as with indicator series

		RMSE between	
	Series	**Quarterly export by Denton's proportional 1st difference (Denton.prop1))**	**Quarterly export by Denton's proportional 2nd difference (Denton.prop2)**
Original (millionUSD)	Quarterly export (export.q)	529.28	772.75
T-1 growth (lag one)%	Quarterly export (export.q)	13.93	15.18
T-4 growth (lag fours)%	Quarterly export (export.q)	11.82	15.98
T-1growth (lag one)%	Quarterly indicator (indicator.q)	6.43	8.33
T-4 growth (lag fours)%	Quarterly indicator (indicator.q)	17.63	21.23

By examining the Table 2, it is detected that the Denton's proportional first difference (RMSE: 529.29) produced more explicit results than the proportional second difference (RMSE: 772.75) with respect to the absolute quarterly export (in million USD). Moreover, on the basis of the the growth rates (T-1 and T-4), Denton's proportional first difference (RMSE: 13.93% and 11.82%) transcended the proportional second difference (RMSE: 15.18% and 15.98%) contrast to original quarterly export. In addition, proportional first difference totally outplayed the proportional second difference as the RMSEs for the T-1 and T-4 growths are 6.43% and 17.63%; and 8.33% and 21.23% respectively relate to the indicator series.

8. DENTON-CHOLETTE'S ADDITIVE VARIANTS

By analysing the outcomes of the series by Denton-Cholette's additive method, It is observed that the actual disaggregated series (depicted in Fig 7) and as well as the T-1 and T-4 growth rates (Fig 8 and Fig 9) of the additive first difference and additive second difference are almost exhibit the same pattern to the original quarterly export although the disperse from the indicator series are quite visible for all the cases.

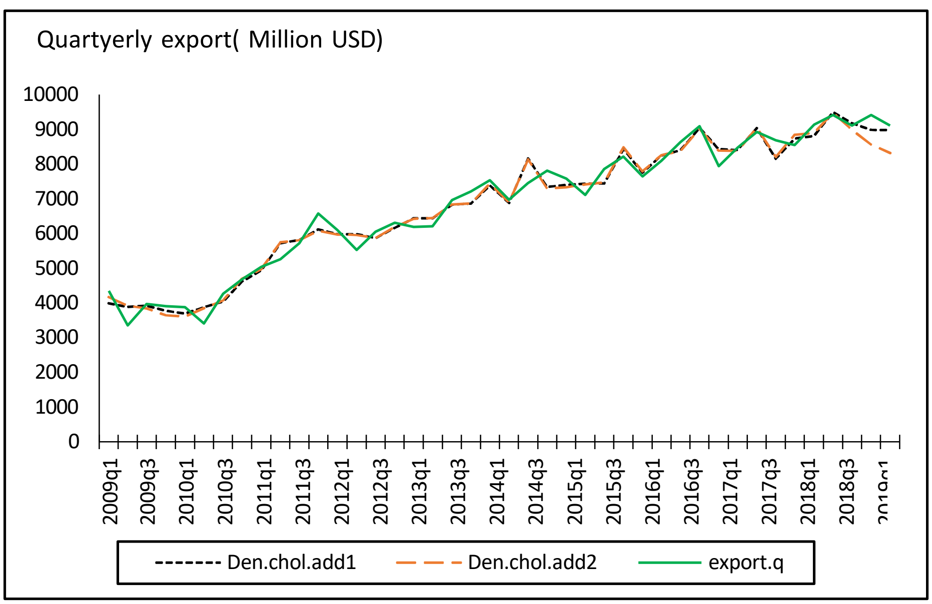

Fig 7. Disaggregated quarterly export by Denton-Cholette additive first and additive second difference, and

actual quarterly exports from 2009q1 to 2019q2

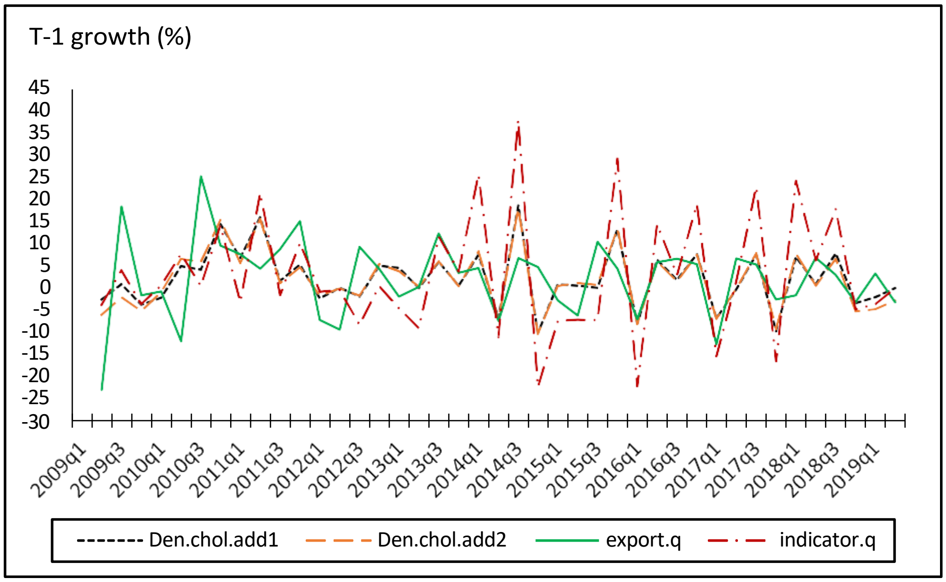

Fig 8. T-1 growth rates (quarterly) of disaggregated quarterly exports by Denton-Cholette additive first and additive second difference, actual quarterly export, and indicator series from 2009q1 to 2019q2

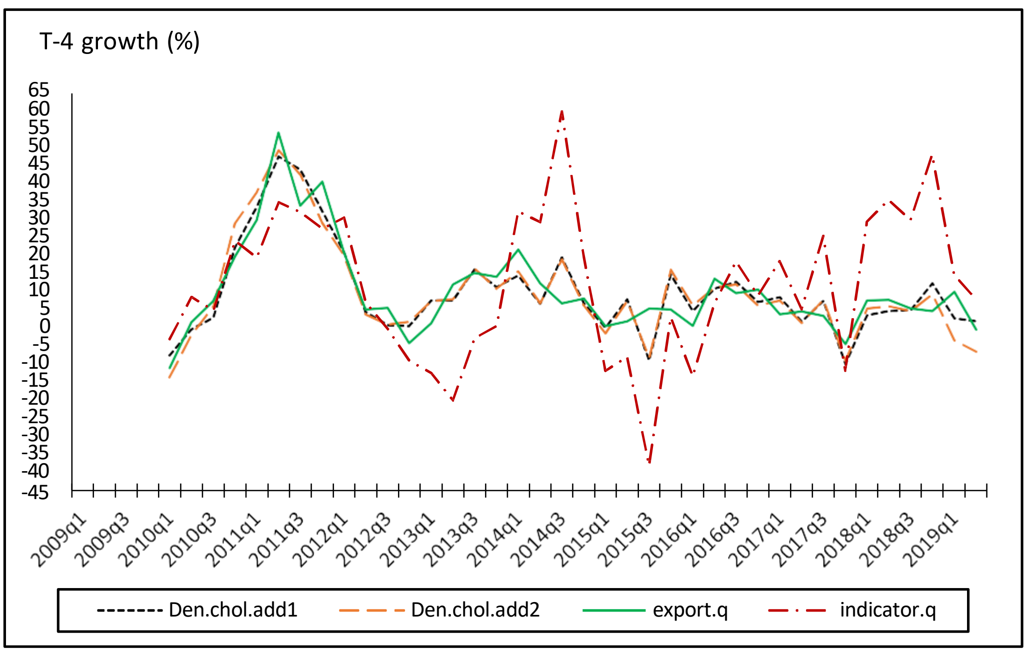

Fig 9. T-4 growth rates (annual) of disaggregated quarterly exports by Denton-Cholette additive first and additive second difference, actual quarterly export, and indicator series from 2009q1 to 2019q2

To reveal further information about the performance of the Denton-Cholette 's additive model, the RMSEs between the actual disaggregated series by additive variants (first and second difference) and the original quarterly export; and as well as with growth rates (T-1 and T-4) of quarterly export and the indicator series are derived and presented in the Table 3.

Table 3. RMSEs between the Denton-Cholette's additive variants and the quarterly export as well as with indicator series

	Series	RMSE between Quarterly export by Denton-Cholette additive 1st difference (Den.chol.add1)	RMSE between Quarterly export by Denton-Cholette additive 2nd difference (Den.chol.add2))
Original (millionUSD)	Quarterly export (export.q)	305.65	345.51
T-1 growth (lag one)%	Quarterly export (export.q)	8.75	8.76
T-4 growth (lag fours)%	Quarterly export (export.q)	5.78	6.25
T-1growth (lag one)%	Quarterly indicator (indicator.q)	8.56	8.58
T-4 growth (lag fours)%	Quarterly indicator (indicator.q)	17.12	17.76

The disaggregation capacity for the case of first and second difference of Denton-Cholette 's additive model are relatively same as the RMSEs of the T-1 and T-4 growth rates in terms of disaggregated series and actual quarterly export and disaggregated series and indicator series are very close. Except for the absolute obtained series; and T-4 growth rates of the disaggregated series and the original quarterly export, the additive first difference approach performs slightly better than the additive second difference approach.

9. DENTON-CHOLETTE 'S PROPORTIONAL VARIANTS

The following Fig 10 shows the pattern of the resulted disaggregated series by Denton-Cholette's proportional first (Den.chol.prop1) and second difference (Den.chol.prop2) in contrast with the original quarterly series of export (export.q), whereas Fig 11 and Fig 12 presented the trend of T-1 and T-4 growth rates of the above series along with indicator series (indicator.q).

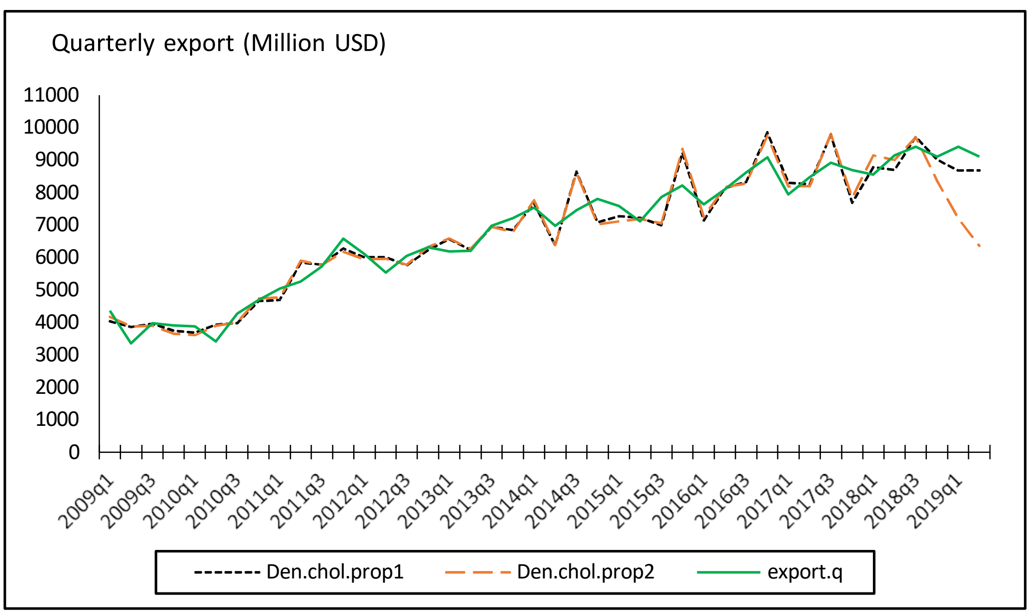

Fig 10. Disaggregated quarterly export by Denton-Cholette proportional first and s proportional second difference, and actual quarterly exports from 2009q1 to 2019q2

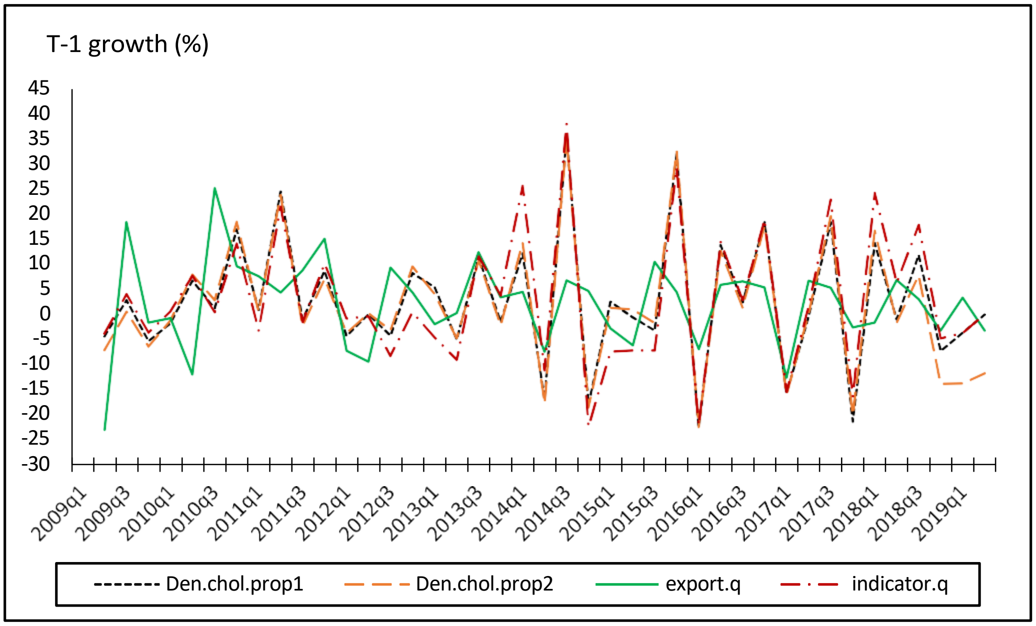

Fig 11. T-1 growth rates (quarterly) of disaggregated quarterly exports by Denton-Cholette proportional first and proportional second difference, actual quarterly export, and indicator series from 2009q1 to 2019q2

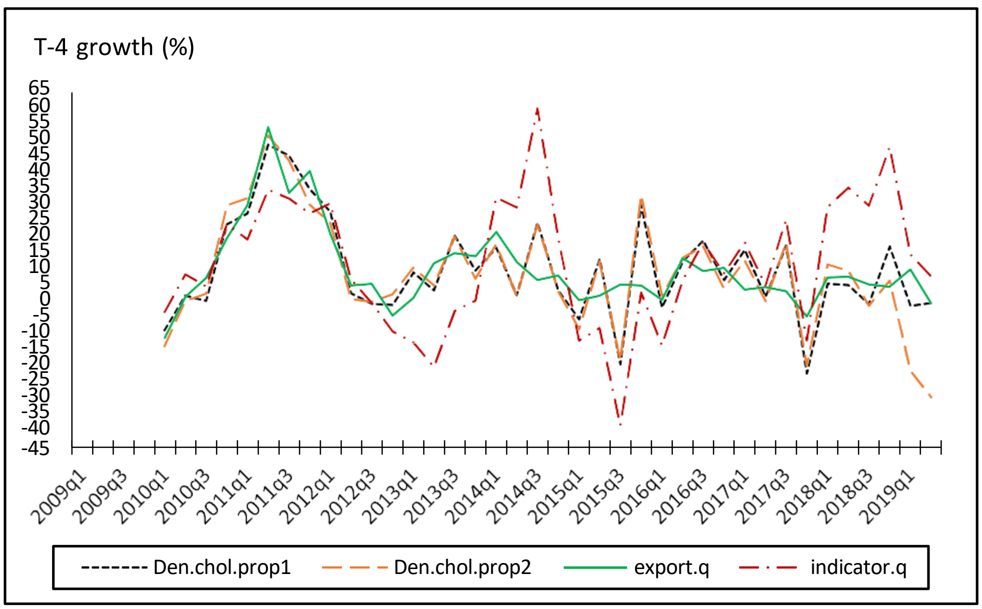

Fig 12. T-4 growth rates (quarterly) of disaggregated quarterly exports by Denton-Cholette proportional first and proportional second difference, actual quarterly exports and indicator series from 2009q1 to 2019q2

From the above Fig 10, it is evident that the disaggregated Denton-Cholette 's proportional first and second difference are very close apart from the end point but both of the resulted series mislead the trend of the original quarterly export. For the T-1 and T-4 growth rates, the proportional first difference following the original series slightly better than the proportional second difference although both the proportional variants are far apart from the indicator series for T-4 growth rate as displayed in Fig 11 and Fig 12. The disaggregation ability of both the proportional variants are judged on the basis of RMSEs are summarized in Table 4.

Table 4. RMSEs between the Denton-Cholette's proportional variants and the quarterly export as well as with indicator series

		RMSE between	
	Series	**Quarterly export by Denton-cholette proportional 1st difference (Den.chol.prop1)**	**Quarterly export by Denton-cholette proportional 2nd difference (Den.chol.prop2)**
Original (millionUSD)	Quarterly export (export.q)	494.66	728.77
T-1 growth (lag one)%	Quarterly export (export.q)	12.86	13.28
T-4 growth (lag fours)%	Quarterly export (export.q)	9.74	11.79
T-1growth (lag one)%	Quarterly indicator (indicator.q)	4.82	5.56
T-4 growth (lag fours)%	Quarterly indicator (indicator.q)	16.73	19.11

On the basis of the RMSEs result (Table 4), it can be said that the Denton-Cholette's proportional first difference variant executes better disaggregated series than the Denton-Cholette's proportional second difference.

10. CONCLUSION

By comparing the root-mean-squared errors (RMSEs), it is summed up that the disaggregation results by the first difference approach is slightly ahead of the second difference for the both of the variants (additive and proportional) of Denton's original; and as well as for Denton-Cholette temporal disaggregation methods (with indicator series) owing to less RMSE than the counterpart. Thus, the result of the RMSEs only for the first difference approach for bath variants in terms of the quarterly exports as well as for indicator series are depicted in Table 5.

Table 5. The RMSEs for first difference for both variants of Denton's original and Denton-Cholette temporal disaggregation methods

	RMSEs				
Methods	Quarterly export (Actual series) (million USD)	Quarterly export T-1 growth (lag one) %	Quarterly export T-4 growth (lag fours) %	Quarterly indicator T-1growth (lag one) %	Quarterly indicator T-4 growth (lag fours) %
Denton's additive 1st difference (Denton.additive1)	357.85	10.37	4.43	9.67	17.90
Denton's proportional 1st difference (Denton.prop1))	529.28	13.93	11.82	6.43	17.63

Denton-Cholette additive 1st difference (Den.chol.add1)	305.65	8.75	5.78	8.56	17.12
Denton-Cholette proportional I st difference (Den.chol.prop1)	494.66	12.86	9.74	4.82	16.73
Lowest	305.65	8.75	4.43	4.82	16.73

By considering the RMSEs between resulted disaggregation series and the original quarterly export (Table 5), first difference of Denton-Cholette additive method contributed the lowest deviation for the actual series as well as for the T-1 growth rates but for the T-4 growth rates, Denton-original additive method outplayed the Denton-Cholette additive method as its produce the least RMSE. On the contrary, first difference of Denton-Cholette proportional variant originated the least variation in terms of the T-1 and T-4 growth rates of the indicator series although for T-4 growth rates, the fluctuations among the different variants are almost same. So, it can be wrapped up and suggested that for disaggregation the yearly series into the quarterly with the help of indicator series, the Denton-Cholette additive method (the first difference) will be more appropriate than the original Denton's additive and proportional as well as Denton-Cholette proportional method.

11. ACKNOWLEDGMENTS

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