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Steady- State Analysis of Heterogeneous Queuing Model with Reverse Balking and Retention of Impatient Customers

Natasha[1], Sapana Sharma[2]

1. Research Scholar, Department of Mathematics and Humanities, MMEC,Maharishi Markandeshwar (Deemed to be University), Mullana- 133203
2. Assistant Professor, Department of Mathematics and Humanities, MMEC,Maharishi Markandeshwar (Deemed to be University), Mullana- 133203

natasharjai@gmail.com, sapanasharma736@gmail.com

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ABSTRACT

In this paper, we develop the concept of reverse balking with reneging and retention in multi-heterogenous servers' queuing system. Reverse balking is one of the latest concepts in queuing theory. It focuses on the behavior of arriving customer who joins the system with high probability in spite of large size of the system and vice-versa. After joining the system, customer can get impatient and leave the system, thus the concept of retention comes to convince them to be remain in queue. Here, steady state solution of the defined system is derived and after that some measures of effectiveness are performed. comparison of the model is carried out. Some special cases of the model are also studied. This type of situation can be seen in investment businesses..

KEYWORDS

Retention, Reneging, Reverse Balking, Heterogeneous, Queuing System.

1. INTRODUCTION

In banks, lines are frequently observed, particularly on Mondays and Fridays. The French word queue originates from the Latin word cauda, which means "tail." In any service system, there will inevitably be customers waiting in line for services; therefore, queue management has been a major difficulty for managers. Thus, it is appropriate to use queuing theory in the financial sector. Since it`s related to lines or queues where clients who can`t be served right away must wait a long time for service, and since time is a resource that needs to be handled well because it`s akin to money. One of the most significant, useful, and possibly most often utilized tools for operational researchers nowadays is queuing theory.

Applications for it can be found in many different domains, such as computers, telecommunications, traffic engineering, and building design for industries, stores, offices, banks, and hospitals.

In many operational scenarios where it is impossible to precisely forecast the rate of client arrivals (or time) and the rate of service facility arrivals (or time), queuing theory can also be utilized.

In queuing theory, "balking" defines the behaviour of customer who arrives but decides not to join the queue. This decision might be based on various factors such as the length of the queue, perceived waiting time, or the current queue conditions. Understanding balking is crucial in modelling and analysing queuing systems to predict customer behaviour and optimize service efficiency. Also, in queuing theory and systems modelling, "reneging" refers to the behaviour of customers who initially join a queue but then leave before being served due to impatience, dissatisfaction with wait time, or other factors. Understanding and mathematically modelling reneging behaviour is essential for optimizing queuing systems and predicting customer behaviour in service environments. The concept of balking and reneging was introduced by Haight [1,2].

In the realm of business, Kumar and Som [11,12] have observed that in larger system size, arriving customer who choose not to join become less and vice-versa. Also, in case of investing field, one can see the change that chances for customers for investing are more even if the size of the system is large. According to Jain et al. [10], this type of customer behaviour is known as "Reverse Balking". For instance, in hospital, where famous and well-educated doctors are giving their services, people get attract towards the fame of that hospital. More number of patients get relief from that hospital attracts other people for treatment even queue size is large

For instance, imagine a group of students considering signing up for an advanced course: Initially, some students decide not to enrol in the course because they perceive it as too challenging or time-consuming. They choose alternative classes that seem easier or more familiar. As the semester progresses, these students observe their peers who are enrolled in the advanced course. They notice that despite the initial perceived difficulty, the course content is captivating, and the learning environment is stimulating. Intrigued by their classmates` experiences and after re-evaluating their priorities, some of these students decide to reverse their initial decision. They choose to drop their current classes and enrol in the advanced course instead, having reconsidered the benefits and their interest in the subject matter.

Reneging has a detrimental effect on the company`s revenue and goodwill since it results in the loss of consumers. The concept of impatient customers` retention was first presented in queue literature by Kumar and Sharma [8]. Retention is the opposite of reneging. It describes customers who stay in the queue after initially deciding to join, despite observing the queue`s conditions or waiting time. A combination of reverse balking, reneging and retention in multiple servers can be modelled mathematically using various stochastic processes. These processes help to analyse the behaviour of customers in a queuing system and how their decisions impact the system. Understanding these behaviour helps in designing and optimizing queue systems to minimize reneging, encourage retention and even predict the likelihood of reverse balking, ultimately aiming to enhance the efficiency and effectiveness of the system. Multiple servers simultaneously cater to customer demands aiming to reduce wait times and enhance service efficiency. The challenge lies in optimizing server utilization while managing customer behaviour like reverse balking, reneging and retention. Also, heterogeneous servers are better than homogeneous servers as there may be possibility of less congestion in queue because of different rate of servers. That's why in the present paper, we examine a queuing model with multiple heterogeneous servers, retention, reneging, and reverse balking.

Remaining content of the paper includes: literature review in section 2, assumptions of the model in section 3, model formulation in mathematics form in section 4, solutions of the equations in steady state in section 5, measurement of effectiveness in section 6, , comparison between two models in section 7, particular cases are included in section 8 and paper is concluded in section 9.

1. LITERATURE REVIEW

In queuing theory, Haight [1] developed the idea of consumers` impatience. He examined the M/M/1/N queue`s steady state behaviour with balking. Haight [2] also noted how a queue having single server behaved with reneging. Ancker and Gafarian [3] worked with balking and reneging on the finite capacity Markovian queue.In [4], he worked on the same idea without reneging. Using the extra variable technique, Rao [5] conducted analysis on a non-Markovian queuing model having single server including balking, interruptions and reneging. In [6], Bae and Kim conducted research on a queue with constant customer patience, and exponential service times for general input having single server.

A finite buffer Markovian queuing system having single server including reneging and balking was the subject of Choudhury and Medhi`s [7]. The study conducted by Kumar and Sharma [8] focused on single and multi-channel queues, reneged consumers, retention and balking. Kumar [9] examines the impatience and retention of consumers in a finite capacity Markovian multi-server queuing model and uses the matrix approach to find the model`s transient solution.

Reverse balking with reverse reneging were used in the M/M/1/N queuing model by Kumar et.al [11,12], who also determined probabilities of the steady state system size. Further retention is added in Kumar et. al [13]. Som and Kumar [14] added 2- heterogeneous servers with reverse balking and reneging to the work of Jain et al. Bounchentouf and Messabhihi [15] conducted an analysis of the heterogeneous 2-server queuing system including reverse balking, reneging, feedback and retention of reneged customer. Kumar and Sharma [16,17] analysed the transient behaviour of M/M/c queuing system with balking and retention of reneging customers. Som [18] worked on the Queuing system having heterogeneous server with feedback, retention of impatient customers but with reverse balking. Kumar and Som [19] studied queuing system having multiple servers with reverse balking and impatient customers.

2. ASSUMPTIONS OF THE MODEL

1. The Poisson process is followed by the customer with the rate of arrival α. The intervals between arrivals are exponentially dispersed and independent.
2. There are various heterogeneous servers (say k). Each server has their own rates to provide service to customers.
3. The system has a limited capacity (say) I.
4. There is a first-come, first-served queuing system.
5. (a) Probability of customers' balk is q' while the system is empty, and opposite to it is probability $p' (= 1 - q')$.

   (b) Probability of customers' balk is $1 - \frac{i}{I-1}$ and opposite to it is probability $\frac{i}{I-1}$ when the system is not empty. Reverse balking is the type of balking that is discussed in (a) and (b). Upon entering the line, every client must wait a while for their service to start.
6. Customer undergoes the process of reneging with rate **ξ**.
7. $q(= 1 - p)$ is the probability of retention of reneged customer.

3. MODEL FORMULATION IN MATHEMATICS FORM

Let the number of customers at time t is i and the probability is denoted as $P_i(t)$:

The C.K equations of the model are:

$$\frac{dP_0(t)}{dt} = -\alpha p' P_0(t) + \mu_1 P_1(t); \qquad i = 0 \qquad (1)$$

$$\frac{dP_1(t)}{dt} = \alpha p' P_0(t) - \left(\mu_1 + \frac{\alpha}{I-1}\right) P_1(t) + (\mu_1 + \mu_2) P_2(t); \qquad i = 1 \qquad (2)$$

$$\frac{dP_i(t)}{dt} = \alpha\left(\frac{i-1}{I-1}\right) P_{i-1}(t) - \left(\sum_{m=1}^{i} \mu_m + \frac{i}{I-1}\alpha\right) P_i(t) + \sum_{m=1}^{i+1} \mu_m P_{i+1}(t); \qquad 2 \le i < k \qquad (3)$$

$$\frac{dP_i(t)}{dt} = \alpha\left(\frac{i-1}{I-1}\right) P_{i-1}(t) - \left\{\sum_{m=1}^{k} \mu_m + \frac{i}{I-1}\alpha + (i-k)\xi p\right\} P_i(t) + \left(\sum_{m=1}^{k} \mu_i + (i-k+1)\xi p\right) P_{i+1}(t); \qquad k \le i \le I-1 \qquad (4)$$

$$\frac{dP_{I-1(t)}}{dt} = \alpha P_{I-1}(t) - \left\{\sum_{m=1}^{k} \mu_m + (I-k)\xi p\right\} P_I(t); \qquad i = I \qquad (5)$$

4. SOLUTION OF THE MODEL IN STEADY STATE

In steady state, model formulation is represented as:

$$0 = -\alpha p' P_0 + \mu_1 P_1; \qquad i = 0 \qquad (6)$$

$$0 = \alpha p' P_0 - \left(\mu_1 + \frac{\alpha}{I-1}\right) P_1 + (\mu_1 + \mu_2) P_2; \qquad i = 1 \qquad (7)$$

$$0 = \alpha\left(\frac{i-1}{I-1}\right) P_{i-1} - \left(\sum_{m=1}^{i} \mu_m + \frac{i}{I-1}\alpha\right) P_i + \sum_{m=1}^{i+1} \mu_m P_{i+1}; \qquad 2 \le i < k \qquad (8)$$

$$0 = \alpha\left(\frac{i-1}{I-1}\right) P_{i-1} - \left\{\sum_{m=1}^{k} \mu_m + \frac{i}{I-1}\alpha + (i-k)\xi p\right\} P_i$$
$$+\left(\sum_{m=1}^{k} \mu_i + (i-k+1)\xi p\right) P_{i+1} \qquad k \le i \le I-1 \quad (9)$$

$$0 = \alpha P_{I-1} - \left\{\sum_{m=1}^{k} \mu_m + (I-k)\xi p\right\} P_I; \qquad i = I \qquad (10)$$

Solving Above (6)- (10) equations, we get

$$P_i = \frac{(i-1)!}{(I-1)^{i-1}} \prod_{c=1}^{i} \frac{\alpha}{\sum_{m=1}^{c} \mu_m} p' P_0; \qquad 1 \le i < k$$

$$= \frac{(i-1)!}{(I-1)^{I-2}} \prod_{c=1}^{k-1} \frac{\alpha}{\sum_{m=1}^{c} \mu_m} \prod_{c=k}^{i} \frac{\alpha}{\left(\sum_{m=1}^{c} \mu_m + (c-k)\xi p\right)} p' P_0; \qquad k \le i \le I-1$$

$$= \frac{(I-2)!}{(I-1)^{I-2}} \prod_{c=1}^{k-1} \frac{\alpha}{\sum_{m=1}^{c} \mu_m} \prod_{c=k}^{i} \frac{\alpha}{\left(\sum_{m=1}^{c} \mu_m + (c-k)\xi p\right)} p' P_0; \qquad i = I$$

Using Normalization condition, $\sum_{i=0}^{I} P_i = 1$, we have

$$P_0 + \sum_{i=1}^{k-1} P_i + \sum_{i=k}^{I-1} P_i + P_I = 1$$

$$P_0 = \left\{ \{1 + \frac{(i-1)!}{(I-1)^{i-1}} \prod_{c=1}^{i} \frac{\alpha}{\sum_{m=1}^{c} \mu_m} p' P_0 + \frac{(i-1)!}{(I-1)^{I-2}} \prod_{c=1}^{k-1} \frac{\alpha}{\sum_{m=1}^{c} \mu_m} \prod_{c=k}^{i} \frac{\alpha}{\left(\sum_{m=1}^{c} \mu_m + (c-k)\xi p\right)} p' P_0 \right.$$
$$\left. + \frac{(I-2)!}{(I-1)^{I-2}} \prod_{c=1}^{k-1} \frac{\alpha}{\sum_{m=1}^{c} \mu_m} \prod_{c=k}^{i} \frac{\alpha}{\left(\sum_{m=1}^{c} \mu_m + (c-k)\xi p\right)} p' P_0 \right\}$$

5. MEASURES OF EFFECTIVENESS

- L_s=Expected System Size (ESS)
- α=Mean Arrival Rate
- R_r=Average Rate of Reneging
- R_t=Average Rate of Retention
- $R_{b'}$=Average Rate of Reverse Balking
- T=System size
- q`=Probability of Reverse Balking having empty system
- q=Probability of Retention of Reneged Customer
- **ξ** =Rate of Reneging
- L_q= Average Queue Length

A. Expected System Size (ESS)

$$L_s = \sum_{i=1}^{I} iP_i$$

$$L_s = \sum_{i=1}^{k-1} iP_i + \sum_{i=k}^{I-1} iP_i + IP_I$$

$$L_s = \sum_{i=1}^{k-1} i\left[\frac{(i-1)!}{(I-1)^{i-1}}\prod_{c=1}^{i}\frac{\alpha}{\sum_{m=1}^{c}\mu_m}\right] p'P_0 + \sum_{i=k}^{I-1} i\left[\frac{(i-1)!}{(I-1)^{I-2}}\prod_{c=1}^{k-1}\frac{\alpha}{\sum_{m=1}^{c}\mu_m}\prod_{c=k}^{i}\frac{\alpha}{(\sum_{m=1}^{c}\mu_m + (c-k)\xi p)}\right]p'P_0$$

$$+ I\left[\frac{(I-2)!}{(I-1)^{I-2}}\prod_{c=1}^{k-1}\frac{\alpha}{\sum_{m=1}^{c}\mu_m}\prod_{c=k}^{i}\frac{\alpha}{(\sum_{m=1}^{c}\mu_m + (c-k)\xi p)}\right]p'P_0$$

B. **Average rate of reneging**:

$$R_r = \sum_{i=k}^{I}(i-k)\xi P_i$$

$$= \sum_{i=k}^{I-1}(i-k)\xi\left[\frac{(i-1)!}{(I-1)^{I-2}}\prod_{c=1}^{k-1}\frac{\alpha}{\sum_{m=1}^{c}\mu_m}\prod_{c=k}^{i}\frac{\alpha}{(\sum_{m=1}^{c}\mu_m+(c-k)\xi p)}\right]p'^{P_0} + (I -$$

$$k)\xi\left[\frac{(I-2)!}{(I-1)^{I-2}}\prod_{c=1}^{k-1}\frac{\alpha}{\sum_{m=1}^{c}\mu_m}\prod_{c=k}^{i}\frac{\alpha}{(\sum_{m=1}^{c}\mu_m+(c-k)\xi p)}\right]p'P_0$$

C. Average rate of reverse balking:

$$R_{b'} = q'\alpha P_0 + \sum_{i=1}^{I-1}\left(1-\frac{i}{I-1}\right)\alpha P_i$$

$$R_{b'} = q'\alpha P_0 + \sum_{i=1}^{k-1}\left(1-\frac{i}{I-1}\right)\alpha\left[\frac{(i-1)!}{(I-1)^{i-1}}\prod_{c=1}^{i}\frac{\alpha}{\sum_{m=1}^{c}\mu_m}\right] p'P_0$$

$$+\sum_{i=k}^{I-1}\left(1-\frac{i}{I-1}\right)\alpha\left[\frac{(i-1)!}{(I-1)^{I-2}}\prod_{c=1}^{k-1}\frac{\alpha}{\sum_{m=1}^{c}\mu_m}\prod_{c=k}^{i}\frac{\alpha}{(\sum_{m=1}^{c}\mu_m + (c-k)\xi p)}\right]p'P_0$$

D. Average rate of retention:

$$R_t = \sum_{i=k}^{I}(i-k)\xi p P_i$$

$$R_r = \sum_{i=k}^{I-1}(i-k)\xi p\left[\frac{(i-1)!}{(I-1)^{I-2}}\prod_{c=1}^{k-1}\frac{\alpha}{\sum_{m=1}^{c}\mu_m}\prod_{c=k}^{i}\frac{\alpha}{(\sum_{m=1}^{c}\mu_m + (c-k)\xi p)}\right]p'P_0 + (I$$

$$- k)\xi p\left[\frac{(I-2)!}{(I-1)^{I-2}}\prod_{c=1}^{k-1}\frac{\alpha}{\sum_{m=1}^{c}\mu_m}\prod_{c=k}^{i}\frac{\alpha}{(\sum_{m=1}^{c}\mu_m + (c-k)\xi p)}\right]p'P_0$$

E. Average Queue Length:

$$L_q = \sum_{i=k}^{I}(i-k)P_i$$

$$= \sum_{i=k}^{I-1}(i-k)\left[\frac{(i-1)!}{(I-1)^{I-2}}\prod_{c=1}^{k-1}\frac{\alpha}{\sum_{m=1}^{c}\mu_m}\prod_{c=k}^{i}\frac{\alpha}{(\sum_{m=1}^{c}\mu_m+(c-k)\xi p)}\right]p'P_0 + (I-k)\left[\frac{(I-2)!}{(I-1)^{I-2}}\prod_{c=1}^{k-1}\frac{\alpha}{\sum_{m=1}^{c}\mu_m}\prod_{c=k}^{i}\frac{\alpha}{(\sum_{m=1}^{c}\mu_m+(c-k)\xi p)}\right]p'P_0$$

6. Comparative Analysis:

[1] Comparison of Average Arrival rate on Expected system size:

In Figure 1 variation of expected system size with respect to arrival rate is compared in two queuing models. Observation shows that expected system size is lower in queuing model having heterogeneous multiple servers with reverse balking and retention of impatient customers in comparison to queuing model having homogeneous multiple servers having reverse balking and impatient customers. This result shows that less congestion in queue appear in case of heterogeneous queuing system. So heterogenous queuing model is better.

Table 1
Impact of α on L_s
When ξ =0.1, k=3, I=10

α	L_{s1}	L_{s2}
5	0.244324	0.179853
6	0.306522	0.22575
7	0.375162	0.277047
8	0.452545	0.335892
9	0.543052	0.40608
10	0.654895	0.494374
11	0.803029	0.612693
12	1.013782	0.781612
13	1.33197	1.035742
14	1.831484	1.431759
15	2.630622	2.060052
16	3.913773	3.061221
17	5.961401	4.648927
18	9.190767	7.140963
19	14.21027	11.00076
20	19.59412	16.89201

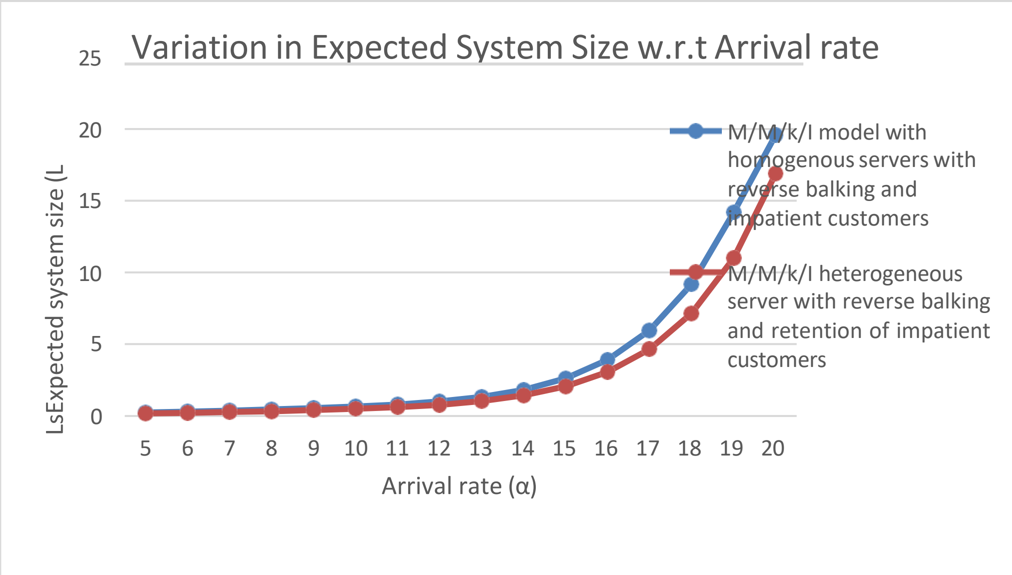


Figure 1: Variation in expected system size w.r.t. arrival rate

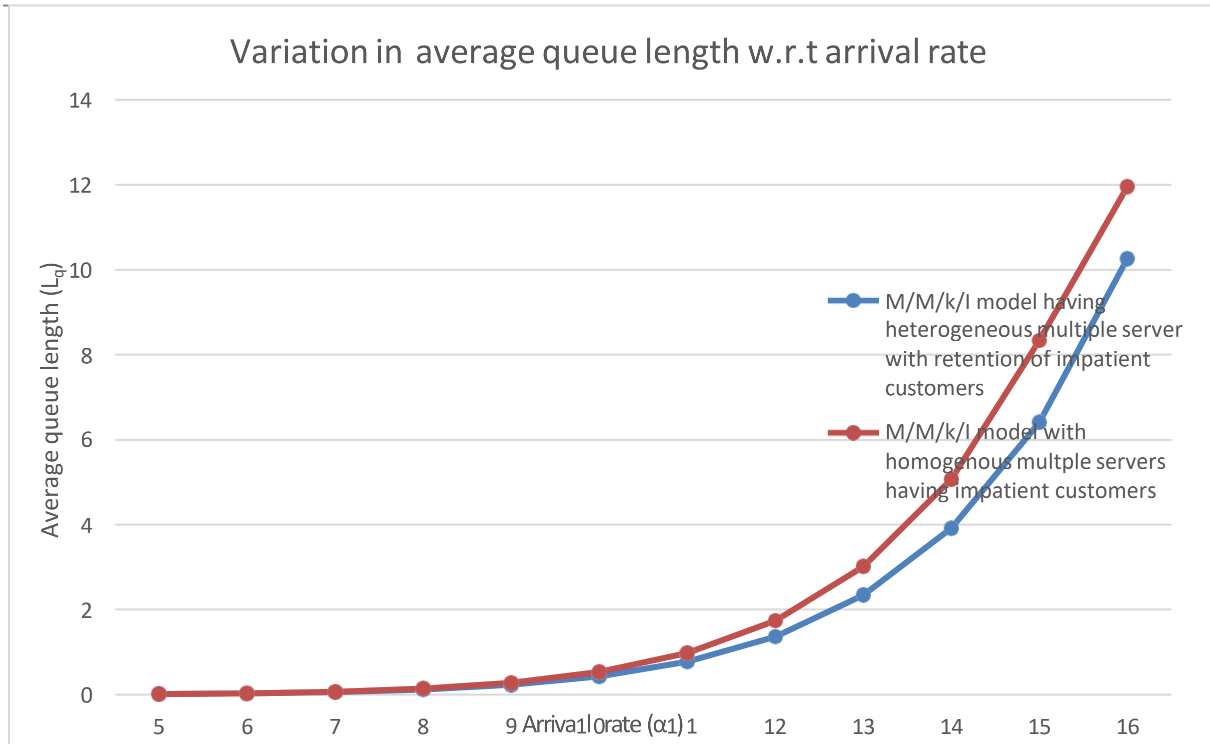
[2] Comparison of Average arrival rate on Average queue length:

In Figure 2 variation of average queue length with respect to arrival rate is compared in two queuing models. Observation shows that average queue length is lower in queuing model having heterogeneous multiple servers with retention of impatient customers in comparison to queuing model having homogeneous multiple servers having impatient customers. This shows that in queuing system having heterogeneous servers customers can get faster service than the other queuing system.

Table 2

Impact of α on L_q
When ξ =0.1, k=3, I=10

α	L_{q1}	L_{q2}
5	0.000217	0.000524
6	0.000712	0.001326
7	0.002079	0.003099
8	0.005522	0.006881
9	0.01358	0.014732
10	0.031285	0.030612
11	0.068108	0.06184
12	0.141079	0.121456
13	0.279602	0.23187
14	0.532656	0.430321
15	0.979248	0.776831
16	1.743241	1.365504
17	3.013908	2.340197
18	5.073888	3.915879
19	8.336556	6.407197
20	11.95974	10.26612



s

Figure 2: Variation in average queue length w.r.t arrival rate

7. PARTICULAR CASES

Case 1:

If q=0, that is, no retention occurs and all servers are homogeneous, then the model reduces to reverse balking having multiple servers with reneging as mentioned in Kumar and Som [19].

Case 2:

If q=0, that is, no retention occurs and k=2, there are two heterogeneous servers, then the model reduces to reverse balking having heterogeneous servers with reneging as mentioned in Som & Kumar [14].

Case 3:

If q=0, that is, no retention occurs, **ξ**=0, that is no reneging occurs, and k=1, that is, there are only one server, then the model reduces to an M/M/1/I model with reverse balking as mentioned in Kumar and Som [11].

8. CONCLUSION

A queuing system with multiple heterogeneous servers, finite capacity, reverse balking, reneging and retention is presented in this paper. Steady state solution and Measures of effectiveness is performed in this paper. Also, comparison in the values of effectiveness between two queuing models is performed which shows that our present model is better than the previous other models and particular cases are also studied in this paper.

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