Library Progress International *Print version ISSN 0970 1052*
Vol.44, No.3, July-December *Online version ISSN 2320 317X*
P. 256040-25645

Original Article *Available online at www.bpasjournals.com*

Enhanced Decision-Making in Warehousing: A Comparative Analysis of Max-Min-Max and Improved (3,2) Fuzzy Composition Methods

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How to cite this article: M.Mary Jansirani,M.Venkatachalapathy,A.Manickam (2024) Enhanced Decision-Making in Warehousing: A Comparative Analysis of Max-Min-Max and Improved (3,2) Fuzzy Composition Methods. *Library Progress International,* 44(3), 256040-25645

ABSTRACT

This paper presents a decision-making approach using a (3,2) fuzzy matrix to evaluate the optimal warehouse for storing products during different seasons, considering multiple criteria such as cost, demand, and storage conditions. The products evaluated include electronics, clothing, food items, furniture, and books, with seasons like spring, summer, autumn, winter, and rainy. We first define a (3,2) fuzzy relation matrix R_0 to evaluate the storage suitability of each product at specific warehouses during various seasons, followed by a matrix R_s to evaluate the suitability of the warehouses in those seasons. Using a max-min-max operation, we compute the composite event relation matrix R_1, which helps identify the optimal warehouse for each product in each season.This approach provides warehouse managers with a systematic and robust method to account for uncertainty in storage decisions.

Key Words: Decision Making; Fuzzy Matrices; Fuzzy Relations; Uncertainty;

AMS subject classifications: 03B05, 03B52, 03E02, 03E72, 03G27.

1. Introduction

Fuzzy matrices are an extension of classical matrix theory, designed to model and manage uncertainty, imprecision, and ambiguity in complex systems. Originating from fuzzy set theory introduced by Lotfi Zadeh in 1965, fuzzy matrices allow for the representation of data where relationships between elements are not precisely defined, but rather characterized by degrees of membership. This makes fuzzy matrices especially useful in situations where crisp, binary decisions (such as true/false or yes/no) are inadequate for capturing the real-world complexity.

In a fuzzy matrix, each element represents the degree to which a certain condition holds true, typically ranging between 0 (completely false) and 1 (completely true). These values, called membership grades, allow for a more flexible and nuanced description of relationships between different entities in the system. Fuzzy matrices have found applications in various fields, including decision-making, control systems, pattern recognition, and data analysis, where uncertainty plays a significant role.

The structure of fuzzy matrices can vary depending on the complexity of the system being modeled. Basic fuzzy matrices involve only a single degree of membership, but more advanced forms, such as interval-valued fuzzy matrices and Fermatean fuzzy matrices, incorporate additional dimensions like non-membership and hesitation. These multi-dimensional matrices, such as (3,2) fuzzy matrices, offer an even richer framework for capturing uncertainty, allowing for more informed and resilient decision-making processes.

Fuzzy matrices are particularly beneficial in decision-making scenarios where multiple criteria must be evaluated, and there is hesitation or uncertainty in assigning definitive values. By incorporating these degrees of uncertainty, fuzzy matrices help decision-makers better manage risk and ambiguity, leading to more robust and comprehensive solutions.

Warehouse selection is a critical aspect of supply chain management, where decisions are often influenced by various factors such as storage conditions, cost-efficiency, demand fluctuations, and seasonal changes. In an increasingly complex environment, managing these factors under conditions of uncertainty or incomplete information becomes a challenge. Traditional decision-making methods may fall short when there is hesitation or conflicting information. To address this, fuzzy set theory has emerged as a powerful tool for handling imprecision in data and providing more flexible decision-making frameworks.

In this context, the use of fuzzy relations, specifically Fermatean fuzzy sets, allows for a more nuanced approach by considering degrees of membership and non-membership. The (3,2) fuzzy relation, a recent extension of fuzzy set theory, provides an advanced model to tackle uncertainty in decision-making. This approach is particularly useful when evaluating multiple warehouses, products, and seasons where the suitability of each option may not be definitively clear.

The proposed methodology provides a systematic framework for decision-making, contributing to a better understanding of warehouse-product-season alignment. This enhances the ability to optimize storage choices, reduce costs, and improve overall supply chain efficiency by leveraging fuzzy relations to account for the inherent uncertainties in warehouse management.

2. Preliminaries

Definition 2.1.

The membership function $\mu_F: U \to [0,1]$ elements of the universe of discourse provide the concept of a fuzzy set. Any value between 0 and 1 qualifies U to be a member of the fuzzy set $0 \leq \mu_F \leq 1$(1 represent the degree of membership of an element U.

Definition 2.2.

let Z be a matrix, $\left[\left(z_{ij}\right)\right]_{u\times v}$, where $z_{ij} \in [0,1]$, $1 \leq i \leq u$ and $1 \leq j \leq v$, then Z is called a fuzzy matrix.

Definition 2.3:

Let X be a nonempty set. The (3,2)-fuzzy set on X is defined to be a structure $G_X = \{\langle x, \mathcal{M}(x), \mathcal{N}(x)\rangle | x \in X\}$, where $\mathcal{M}: X \to [0,1]$ is the degree of membership of x to G and $\mathcal{N}: X \to [0,1]$ is the degree of non-membership of x to G such that $0 \leq \left(\mathcal{M}(x)\right)^3 + \left(\mathcal{N}(x)\right)^2 \leq 1$.

Definition 2.4:

Let δ be a positive real number ($\delta > 0$). If $\mathcal{D}_1 = (\kappa_{\mathcal{D}_1}, \tau_{\mathcal{D}_1})$ and $\mathcal{D}_2 = (\kappa_{\mathcal{D}_2}, \tau_{\mathcal{D}_2})$ are two (3, 2)-Fuzzy Sets, then their operations are defined as follows:

i. $\mathcal{D}_1 \cup \mathcal{D}_2 = \{\min\left(\kappa_{\mathcal{D}_1}, \kappa_{\mathcal{D}_2}\right), \max\left(\tau_{\mathcal{D}_1}, \tau_{\mathcal{D}_2}\right)\}$

ii. $\mathcal{D}_1 \cap \mathcal{D}_2 = \{\max\left(\kappa_{\mathcal{D}_1}, \kappa_{\mathcal{D}_2}\right), \min\left(\tau_{\mathcal{D}_1}, \tau_{\mathcal{D}_2}\right)\}$

iii. $\mathcal{D}_1^c = (\tau_{\mathcal{D}_1}, \kappa_{\mathcal{D}_1})$

Definition 2.5:

Let $Q(X \longrightarrow Y)$ and $R(Y \longrightarrow Z)$ be two (3, 2)-Fuzzy relations. Then, for all $(m, r) \in X \times Z$ and $n \in Y$,

(i) The max-min-max composition $R \circ Q$ is the (3, 2)-fuzzy relation from X to Z defined by

$$\kappa_{R\circ Q}(m,r) = \max\{\min(\kappa_R(m.n), \kappa_Q(\text{n,r})\}$$

$$\tau_{R\circ D}(m,r) = \min\{\max(\tau_R(m.n), \tau_Q(n,r)\}$$

(i) The improved composite relation $R \circ Q$ is the (3, 2)-fuzzy relation from X to Z such that

$$\kappa_{R\circ Q}(m,r) = \max\left\{\frac{\kappa_R(m.n) + \kappa_Q(\text{n,r})}{2}\right\}$$

$$\tau_{R\circ Q}(m,r) = \min\left\{\frac{(\tau_R(m.n) + \tau_Q(n,r)\}}{2}\right\}$$

3.Algorithm

Step 1: Define the sets

Step 2: Enter the (3,2) fuzzy relation matrix$R_0$for seasons and products

Step 3: Enter the (3,2) fuzzy relation R_s indicates the Relation for Warehouse and Season Suitability

Step 4: Compute the **event relation** matrix $R_1 = R_0 \circ R_s$ by using max-min-max composition and improved max-min-max composition.

Step 5: Final conclusion is based on highest membership obtained from the set.

3.1 Numerical Example:

This example demonstrates how an (3,2) fuzzy matrix can help make a decision when multiple criteria are involved, and there is some level of uncertainty or hesitation in evaluations.

In this example, we consider:

1. **Warehouses** (W): Set of five warehouses, $W = \{w_1, w_2, w_3, w_4, w_5\}$.
2. **Products** (P): Set of five products, $P = \{p_1, p_2, p_3, p_4, p_5\}$.
3. **Seasons** (S): Set of five seasons, $S = \{s_1, s_2, s_3, s_4, s_5\}$ which are spring, summer, autumn, winter, and rainy.
4. **Warehousing Decision**: We aim to determine the optimal warehouse for storing a particular product during a particular season based on factors like cost, demand, and storage conditions.

Step 1: Define the Sets

- **Products (P)**: $P = \{p_1 = electronics, p_2 = clothing, p_3 = fooditems, p_4 = furniture, p_5 = books\}$.
- **Seasons (S)**: $S = \{s_1 = rainy, s_2 = winter, s_3 = spring, s_4 = summer, s_5 = autumn\}$
- **Warehouses (W)**: $W = \{w_1, w_2, w_3, w_4, w_5\}$.

Step 2: (3,2) Fuzzy Relation Matrix for seasons and products

We create a matrix R_0 representing the (3,2) fuzzy relation for storing each product at a specific warehouse across different seasons. Each cell contains an (3,2) fuzzy number represented as a tuple $(\grave{\mu}, \dot{\nu})$ where:

$s_1 s_2 s_3 s_4 s_5$

$$R_0 = \begin{matrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{matrix} \begin{bmatrix} (0.81, 0.23) & (0.98, 0.75) & (0.36, 0.43) & (0.26, 0.54) & (0.68, 0.79) \\ (0.79, 0.15) & (0.49, 0.19) & (0.68, 0.29) & (0.48, 0.23) & (0.12, 0.25) \\ (0.88, 0.05) & (0.36, 0.88) & (0.72, 0.82) & (0.84, 0.45) & (0.56, 0.78) \\ (0.34, 0.68) & (0.56, 0.45) & (0.77, 0.37) & (0.92, 0.23) & (0.45, 0.23) \\ (0.43, 0.98) & (0.56, 0.32) & (0.14, 0.85) & (0.60, 0.10) & (0.56, 0.98) \end{bmatrix}$$

Step 3: (3,2) Fuzzy Relation for Warehouse and Season Suitability

Next, we define a relation R_s, which represents the (3,2) fuzzy relation for the suitability of storing any product at a specific warehouse across different season

$$w_1 w_2 w_3 w_4 w_5$$

$$R_s = \begin{matrix} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \end{matrix} \begin{bmatrix} (0.66, 0.11) & (0.65, 0.15) & (0.63, 0.09) & (0.69, 0.14) & (0.55, 0.79) \\ (0.59, 0.65) & (0.39, 0.29) & (0.78, 0.29) & (0.88, 0.73) & (0.42, 0.25) \\ (0.98, 0.85) & (0.46, 0.82) & (0.79, 0.87) & (0.58, 0.45) & (0.96, 0.48) \\ (0.94, 0.28) & (0.16, 0.44) & (0.17, 0.37) & (0.02, 0.23) & (0.65, 0.23) \\ (0.33, 0.98) & (0.16, 0.92) & (0.14, 0.85) & (0.20, 0.70) & (0.76, 0.68) \end{bmatrix}$$

Step 4: Decision-Making Based on (3,2) Fuzzy Relations

Firstly we compute the **event relation** matrix $R_1 = R_0 \circ R_s$, using an max-min-max operation for fuzzy matrix. This operation combines the degrees of membership and non-membership for a more comprehensive decision-making process.

The resulting matrix is given by:

$$R_1 = R_0 \circ R_s,$$

$$s_1 s_2 s_3 s_4 s_5$$

$$R_1 = \begin{matrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{matrix} \begin{bmatrix} (0.81, 0.23) & (0.98, 0.75) & (0.36, 0.43) & (0.26, 0.54) & (0.68, 0.79) \\ (0.79, 0.15) & (0.49, 0.19) & (0.68, 0.29) & (0.48, 0.23) & (0.12, 0.25) \\ (0.88, 0.05) & (0.36, 0.88) & (0.72, 0.82) & (0.84, 0.45) & (0.56, 0.78) \\ (0.34, 0.68) & (0.56, 0.45) & (0.77, 0.37) & (0.92, 0.23) & (0.45, 0.23) \\ (0.43, 0.98) & (0.56, 0.32) & (0.14, 0.85) & (0.60, 0.10) & (0.56, 0.98) \end{bmatrix}$$

$\circ$

$$w_1 w_2 w_3 w_4 w_5$$

$$\begin{matrix} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \end{matrix} \begin{bmatrix} (0.66, 0.11) & (0.65, 0.15) & (0.63, 0.09) & (0.69, 0.14) & (0.55, 0.79) \\ (0.59, 0.65) & (0.39, 0.29) & (0.78, 0.29) & (0.88, 0.73) & (0.42, 0.25) \\ (0.98, 0.85) & (0.46, 0.82) & (0.79, 0.87) & (0.58, 0.45) & (0.96, 0.48) \\ (0.94, 0.28) & (0.16, 0.44) & (0.17, 0.37) & (0.02, 0.23) & (0.65, 0.23) \\ (0.33, 0.98) & (0.16, 0.92) & (0.14, 0.85) & (0.20, 0.70) & (0.76, 0.68) \end{bmatrix}$$

$$w_1 w_2 w_3 w_4 w_5$$

$$R_1 = \begin{matrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{matrix} \begin{bmatrix} (0.66, 0.23) & (0.65, 0.23) & (0.78, 0.23) & (0.88, 0.23) & (0.68, 0.48) \\ (0.68, 0.15) & (0.65, 0.15) & (0.68, 0.15) & (0.69, 0.15) & (0.68, 0.23) \\ (0.84, 0.11) & (0.65, 0.15) & (0.72, 0.09) & (0.58, 0.14) & (0.72, 0.45) \\ (0.92, 0.28) & (0.46, 0.44) & (0.77, 0.37) & (0.58, 0.23) & (0.77, 0.23) \\ (0.60, 0.28) & (0.43, 0.32) & (0.56, 0.32) & (0.56, 0.23) & (0.60, 0.23) \end{bmatrix}$$

By using the above matrix we conclude that, product p_1 and p_2 is suitable to store in w_4, p_3 and p_4 is suitable to store in w_1 and p_5 is suitable to store in w_1 and w_5.

Next we now compute the **event relation** matrix $R_1 = R_0 \circ R_s$, using an improved max-min-max operation for fuzzy matrix.

$$w_1 w_2 w_3 w_4 w_5$$

$$R_1 = \begin{matrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{matrix} \begin{bmatrix} (0.785, 0.17) & (0.73, 0.19) & (0.88, 0.16) & (0.93, 0.185) & (0.72, 0.385) \\ (0.83, 0.13) & (0.72, 0.15) & (0.735, 0.12) & (0.74, 0.145) & (0.82, 0.23) \\ (0.89, 0.08) & (0.765, 0.1) & (0.755, 0.07) & (0.785, 0.095) & (0.84, 0.34) \\ (0.93, 0.255) & (0.615, 0.335) & (0.78, 0.3) & (0.72, 0.23) & (0.865, 0.165) \\ (0.77, 0.19) & (0.54, 0.27) & (0.67, 0.235) & (0.72, 0.165) & (0.66, 0.19) \end{bmatrix}$$

By using the above matrix we conclude that, product p_1,is suitable to store in w_4, p_2, p_3, p_4 and p_5 are suitable to store in w_1.

Step 5: Decision Making

In the standard max-min-max composition, the resulting event relation matrix R_1 provides a general view of warehouse suitability, indicating that products p_1 and p_2 are suitable for storage in warehouse w_4, while products p_3, p_4 and p_5 are best stored in warehouse w_1 and w_5. However, the improved max-min-max composition enhances this by refining the membership and non-membership degrees, offering more nuanced results. For instance, product $p_1$is still best suited for warehouse w_4, but products p_2,p_3, p_4 and $p_5$are now all found to be more suitable for storage in warehouse w_1.The improved composition provides a more detailed and accurate reflection of the interactions between product and warehouse suitability, allowing for more confident decision-making. It reduces uncertainty and hesitation in evaluations, leading to more precise and reliable outcomes for product allocation.

4.Conclusion

The (3,2) fuzzy matrix model effectively handles uncertainty and hesitation in multi-criteria decision-making scenarios. In this example, by considering both product and seasonal conditions, the fuzzy matrix approach was able to identify optimal warehouse locations for various products. The final matrix using an improved max-min-max operation highlights the importance of using advanced fuzzy relation techniques in decision-making. The approach demonstrated its potential to aid in efficient product allocation across seasons, contributing to better warehousing decisions. This model allows them to account for uncertainty and conflicting information, providing a more comprehensive understanding of the storage environment and requirements.

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