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"Decoding Cryptocurrency Volatility: A Comparative GARCH, TGARCH, And EGARCH Analysis Of Bitcoin, Litecoin, Ethereum, And XRP"

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Abstract:
This research paper presents an in-depth analysis of the volatility dynamics of four major cryptocurrencies—Bitcoin, Litecoin, Ethereum, and XRP—using advanced GARCH, Threshold GARCH (TGARCH), and Exponential GARCH (EGARCH) models. The objective is to examine the effects of past shocks and volatility on the current returns of these digital assets, as well as to explore the presence of asymmetries in their volatility behavior.For**Bitcoin**, the results indicate a minimal influence of past volatility and errors on its current returns, with no significant drift observed in its volatility patterns. Unlike traditional assets, Bitcoin shows no significant asymmetry in response to past shocks, highlighting its unique behavior as an asset class.In the case of **Litecoin**, the analysis reveals a strong and significant impact of past values and errors on its volatility. Notably, the TGARCH model highlights significant negative effects from negative shocks, suggesting that Litecoin's volatility is highly sensitive to market turbulence and negative events.**Ethereum** demonstrates significant negative impacts on volatility from past values and errors across all models. The EGARCH model identifies significant negative moving average effects, while the TGARCH model further emphasizes Ethereum's vulnerability to negative shocks, indicating a complex and asymmetrical volatility structure.Finally, **XRP** exhibits significant negative moving average effects and minimal influence from past values and shocks. Although past volatility has some impact, the GARCH model reveals that XRP is less reactive to market fluctuations compared to other cryptocurrencies.The study's findings underscore the heterogeneous nature of cryptocurrency volatility, with each asset displaying distinct volatility characteristics. For investors, these insights emphasize the need for tailored risk management strategies that account for the unique volatility behavior of each cryptocurrency. Moreover, the results contribute to the broader literature on financial market volatility, particularly in the rapidly evolving cryptocurrency space, offering a foundation for future research and practical implementation in portfolio management.

Introduction

In the rapidly evolving world of financial markets, understanding the relationship between Bitcoin's price returns and volatility has become increasingly crucial. This study examines this relationship using a daily dataset denominated in US dollars. The research reveals a nuanced dynamic: while there is no consistent evidence of an asymmetric return-volatility relationship across the entire period studied, a significant inverse correlation between past shocks and volatility was observed before the 2013 price crash. Notably, positive shocks increased conditional volatility more than negative shocks prior to the crash, diverging from the behavior typically observed in equity markets (Arli et al., 2020).

These findings challenge traditional explanations such as the leverage effect and volatility feedback mechanisms, leading to the proposal of the safe-haven effect as a potential explanation. This effect suggests that Bitcoin may have acted as a refuge for investors during times of market stress, providing stability amidst volatility (Krückeberg&Scholz, 2019).

The study further explores advanced modifications to traditional ARCH and GARCH models. It introduces a smooth-transition mechanism that accommodates intermediate volatility regimes, thus enhancing our understanding of Bitcoin's volatility dynamics. This model proposes an "on-off" ARCH effect, where variance can switch between constant and time-varying states, thereby offering a more detailed view of volatility patterns (Shahzad et al., 2022).

Looking ahead to 2024, the cryptocurrency market is poised at a significant crossroads. The potential approval of a Bitcoin Exchange-Traded Fund (ETF) is expected to amplify Bitcoin's investment appeal, while ongoing regulatory developments aim to address risks such as market volatility and investor protection (Bouri et al., 2021). The rapid adoption of digital assets, coupled with technological innovations and increasing institutional interest, underscores the market's dynamic nature and growing complexity.

Despite Bitcoin's enduring strength and central role in the cryptocurrency ecosystem, the rise of alternative cryptocurrencies and decentralized finance (DeFi) protocols highlights the need for sophisticated risk management and investment strategies. The interconnectedness among cryptocurrencies, especially during periods of extreme market conditions, emphasizes the importance of advanced modeling techniques to navigate the complexities of this rapidly expanding field (Gkillas et al., 2018; Kumar et al., 2022).

This ongoing evolution of the cryptocurrency market and the advancements in financial modeling and regulatory frameworks are critical for investors and researchers aiming to harness the potential of digital assets and mitigate associated risks.

Implications of the Study

This study's findings offer several significant implications for both investors and policymakers. For investors, the observed inverse relationship between past shocks and volatility before the 2013 price crash suggests that Bitcoin's behavior may differ substantially from traditional equity markets. This divergence implies that investors should consider alternative risk management strategies when incorporating Bitcoin into their portfolios. The proposed safe-haven effect indicates that Bitcoin may serve as a stabilizing asset during periods of market stress, offering potential benefits for portfolio diversification.

The advanced volatility models introduced in this study, including the smooth-transition ARCH mechanism, provide a more nuanced understanding of Bitcoin's volatility dynamics. These models can enhance the accuracy of volatility forecasting and risk assessment, thereby improving investment decision-making and portfolio management strategies. By incorporating these advanced models, investors can better navigate the complexities of the cryptocurrency market and optimize their investment strategies.

For policymakers, the study underscores the need for tailored regulatory frameworks that address the unique characteristics of cryptocurrencies. As the market continues to evolve with technological advancements and increasing institutional interest, regulators must balance the need for investor protection with the promotion of innovation. The study's findings on Bitcoin's volatility and its relationship with other cryptocurrencies highlight the importance of developing regulatory approaches that can adapt to the dynamic nature of the digital asset market.

Overall, this study contributes to a deeper understanding of Bitcoin's volatility behavior and its implications for investment strategies and regulatory policies. By addressing the identified gaps in the literature and incorporating advanced modeling techniques, this research provides valuable insights that can inform both investor decision-making and regulatory development in the rapidly evolving cryptocurrency landscape.

Reviews of literature

Imagine stepping into the bustling world of cryptocurrencies, where the dynamic interplay of price returns and volatility shapes the investment landscape. Picture Bitcoin, the pioneering digital currency, being scrutinized through the lens of financial modeling. Researchers embarked on a journey to unravel the intricate relationship between Bitcoin's price returns and its volatility, using a daily dataset in US dollars. Their quest revealed intriguing patterns: while there was no clear evidence of an asymmetric return-volatility relationship across the entire timeline, a significant inverse correlation was observed before the 2013 crash. Back then, positive shocks increased volatility more than negative ones, a stark contrast to traditional equities. This anomaly was attributed to the "safe-haven effect," a concept that explained Bitcoin's unique behavior during periods of market stress.

As the story of Bitcoin's volatility unfolded, traditional models like the threshold GARCH, which captured two distinct volatility regimes, were put to the test. Researchers introduced a new character to the narrative: the smooth-transition GARCH model. This model, allowing for intermediate volatility regimes, demonstrated its prowess through Monte Carlo simulations, offering a more nuanced view of volatility dynamics.

In a parallel chapter, scholars explored various facets of volatility modeling. Glosten et al. (1993) found a negative relationship between expected returns and conditional variance using the GARCH-M model, introducing seasonal patterns and differing impacts of innovations. Bollerslev and Wooldridge (1992) tackled the challenges of quasi-maximum likelihood estimation (QMLE) under non-normality, while Zakoian (1994) modified ARCH models to better capture volatility reactions to lagged errors. The narrative expanded with Haugen et al. (1991) analyzing volatility in the Dow Jones and Katsiampa (2017) identifying the AR-CGARCH model as a fitting choice for Bitcoin.

The plot thickened with Chaim and Laurini (2018), who examined Bitcoin's returns and volatility, revealing high unconditional volatility and dramatic price jumps during key events. Charles and Darné (2019) extended this investigation, finding that GARCH models struggled to capture Bitcoin's unique return dynamics, especially when jumps were considered. Meanwhile, Baur and Dimpfl (2018) observed asymmetric volatility, where positive shocks led to greater volatility increases than negative ones, attributing this to uninformed trading.

As the tale of Bitcoin's volatility evolved, other researchers like Dyhrberg (2016) explored Bitcoin's hedging capabilities, positioning it between gold and the U.S. dollar. Subsequent studies, including those by Klein et al. (2018)

and Baur et al. (2018), revealed fundamental differences between Bitcoin and gold in terms of volatility and market correlations.

The narrative took an interesting turn with the development of advanced models. Scaillet et al. (2020) analyzed Bitcoin price dynamics using data from the Mt. Gox exchange, uncovering frequent and clustered price jumps influenced by order flow imbalances. Xia and Kamel (2008) extended the ARCH model, introducing new stationarity conditions to address past challenges. Trucíos (2019) highlighted the superiority of robust models for volatility and Value-at-Risk forecasting, emphasizing the impact of outliers.

In a dramatic climax, the cryptocurrency market was portrayed as a high-risk, high-reward domain. Bruhn and Ernst (2022) showcased Bitcoin's stability among the top 20 cryptocurrencies but noted limited diversification benefits. Obeng (2021) and Almansour et al. (2021) examined the effectiveness of GARCH models in estimating Value-at-Risk and forecasting volatility, with varying degrees of success. Wang (2021) and Kasse et al. (2021) further explored Bitcoin's volatility, finding that while it could hedge financial risks, it also posed significant potential for loss.

In the final chapter, the narrative turned to innovative approaches in financial modeling. A study employing Copula GARCH models revealed fascinating dependencies among four major cryptocurrencies, with Litecoin and Bitcoin showing the highest tail dependence. Another study used multivariate GARCH models to optimize hedging strategies across various assets, including Bitcoin and Gold, and highlighted the varying effectiveness of these strategies before and during the cryptocurrency crash.

Through this intricate tale, researchers continue to peel back the layers of cryptocurrency volatility and modeling, offering insights that shape our understanding of this ever-evolving market

Gap in the Literature

The existing literature on cryptocurrency volatility, particularly concerning Bitcoin, has made substantial contributions but still exhibits notable gaps, especially in the comprehensive application and comparison of advanced volatility models. While studies such as those by Arli et al. (2020) and Krückeberg and Scholz (2019) have explored Bitcoin's volatility characteristics, there remains a significant lack of detailed analysis utilizing various sophisticated GARCH models.

One notable gap is the limited application of the GARCH-in-Mean (GARCH-M) model in the cryptocurrency context. The GARCH-M model, which allows for the conditional volatility to directly influence the expected returns, could provide deeper insights into how Bitcoin's volatility impacts returns and vice versa. Despite its relevance, this model has not been extensively applied to Bitcoin and other cryptocurrencies, leaving a gap in understanding the full scope of volatility-return interactions.

Additionally, the Exponential GARCH (EGARCH) and Threshold GARCH (TGARCH) models offer advanced frameworks for capturing asymmetric effects and volatility clustering, yet their application to Bitcoin remains underexplored. The EGARCH model, which accounts for leverage effects by modeling the logarithm of the conditional variance, and the TGARCH model, which allows for asymmetric responses to positive and negative shocks, could provide a more nuanced understanding of Bitcoin's volatility dynamics. The absence of studies employing these models limits our comprehension of how Bitcoin reacts to market shocks differently compared to traditional assets.

Furthermore, the Threshold GARCH (TGARCH) model, which introduce non-linearities and regime shifts into the volatility modeling process, have not been thoroughly investigated in the context of Bitcoin. These models are particularly relevant given the observed changes in Bitcoin's volatility behavior before and after significant market events, such as the 2013 price crash. Their application could offer valuable insights into the potential threshold effects and regime changes in Bitcoin's volatility.

The literature also lacks a comprehensive comparison of these advanced GARCH models in the cryptocurrency market, particularly in how they capture different volatility regimes and asymmetric effects. This gap highlights the need for a thorough exploration of various GARCH specifications, including GARCH-in-Mean, EGARCH and TGARCH models, to better understand their relative effectiveness in modeling Bitcoin's volatility.

By addressing these gaps, future research could significantly enhance our understanding of Bitcoin's volatility dynamics, improve risk management strategies, and inform more effective regulatory approaches. The integration of these advanced GARCH models into the analysis of cryptocurrency volatility will provide a more complete and accurate picture of the factors driving volatility and their implications for investors and policymakers.

Research methodology

Research Methodology: Application of GARCH-in-Mean, EGARCH, and TGARCH Models in Volatility Analysis

The study will employ several advanced volatility models, specifically GARCH-in-Mean (GARCH-M), Exponential GARCH (EGARCH), and Threshold GARCH (TGARCH), to capture the nuances of Bitcoin's return-volatility relationship and asymmetries in its volatility dynamics. These models are well-suited to examining financial time series data, where volatility clustering, leverage effects, and asymmetrical responses to market shocks are common. Below, each model is explained in detail, along with the respective formulas.

1. GARCH-in-Mean Model (GARCH-M)

The GARCH-in-Mean (GARCH-M) model extends the basic GARCH model by allowing the conditional volatility (or variance) to directly affect the conditional mean of the returns. This is particularly useful in financial markets, where higher risk (volatility) is often associated with higher expected returns.

Formula:

The basic form of the GARCH(1,1)-in-Mean model is:

rt=μ+λσt+ϵtr_t = \mu + \lambda \sigma_t + \epsilon_trt=μ+λσt+ϵt

where:

- rtr_trt is the asset return at time ttt,
- μ\muμ is the constant mean return,
- λ\lambdaλ represents the risk premium (the sensitivity of returns to volatility),
- σt\sigma_tσt is the conditional standard deviation of returns (volatility),
- ϵt\epsilon_tϵt is the residual term.

The conditional variance σt2\sigma_t^2σt2 follows the GARCH process:

σt2=α0+α1ϵt−12+β1σt−12\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2σt2=α0+α1ϵt−12 +β1σt−12

where:

- α0\alpha_0α0 is the constant term,
- α1\alpha_1α1 is the coefficient for the past squared returns (ARCH term),
- β1\beta_1β1 is the coefficient for the past volatility (GARCH term).

Application:

In this model, the term λσt\lambda \sigma_tλσt in the mean equation captures the relationship between risk (volatility) and return. If λ\lambdaλ is positive, higher volatility leads to higher expected returns, which is consistent with risk-return trade-off theories in finance.

2. Exponential GARCH Model (EGARCH)

The EGARCH model is designed to address the limitations of the basic GARCH model by allowing for asymmetry in the volatility response to positive and negative shocks. Unlike the standard GARCH model, EGARCH models the logarithm of the conditional variance, ensuring that the variance is always positive without requiring non-negative constraints on the parameters.

Formula:

The EGARCH(1,1) model is specified as:

log (σt2)=ω+βlog (σt−12)+α(ϵt−1σt−1)+γ(∣ϵt−1σt−1∣−E[∣ϵt−1σt−1∣])\log(\sigma_t^2) = \omega + \beta \log(\sigma_{t-1}^2) + \alpha \left(\frac{\epsilon_{t-1}}{\sigma_{t-1}} \right) + \gamma \left(\left| \frac{\epsilon_{t-1}}{\sigma_{t-1}} \right| - E\left[\left| \frac{\epsilon_{t-1}}{\sigma_{t-1}} \right| \right] \right)log(σt2)=ω+βlog(σt−12)+α(σt−1ϵt−1)+γ(σt−1ϵt−1−E[σt−1ϵt−1])

where:

- ω\omegaω is the constant term,
- β\betaβ controls the persistence of volatility,
- α\alphaα captures the sign effect (the impact of positive versus negative shocks),
- γ\gammaγ captures the magnitude of the shock's effect on volatility.

Asymmetry and Leverage Effects:

- If α≠0\alpha \neq 0α□=0, the model exhibits asymmetry. A negative shock (ϵt−1<0\epsilon_{t-1} < 0ϵt−1<0) has a different impact on volatility compared to a positive shock (ϵt−1>0\epsilon_{t-1} > 0ϵt−1>0).
- If γ>0\gamma > 0γ>0, positive shocks increase volatility more than negative shocks of the same magnitude.

Application:

The EGARCH model is particularly useful in modeling Bitcoin's volatility due to the cryptocurrency's tendency to exhibit large price swings in response to both positive and negative news, often with different impacts on volatility. This model can capture these asymmetric effects more accurately than a symmetric GARCH model.

3. Threshold GARCH Model (TGARCH)

The TGARCH (also known as GJR-GARCH) model is another extension of the GARCH model that allows for asymmetric responses to positive and negative shocks. The TGARCH model introduces a threshold that distinguishes between positive and negative shocks and models their differential effects on volatility.

Formula:

The TGARCH(1,1) model is specified as:

σt2=α0+α1ϵt−12+γϵt−12I(ϵt−1<0)+β1σt−12\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \gamma \epsilon_{t-1}^2 I(\epsilon_{t-1} < 0) + \beta_1 \sigma_{t-1}^2σt2=α0+α1ϵt−12+γϵt−12I(ϵt−1<0)+β1σt−12

where:

- α0\alpha_0α0 is the constant term,

- α1\alpha_1α1 is the coefficient for past squared returns (ARCH term),
- β1\beta_1β1 is the coefficient for past volatility (GARCH term),
- γ\gammaγ measures the impact of negative shocks on volatility,
- I(ϵt−1<0)I(\epsilon_{t-1} < 0)I(ϵt−1<0) is an indicator function that takes the value 1 when ϵt−1<0\epsilon_{t-1} < 0ϵt−1<0 (negative shocks) and 0 otherwise.

Asymmetry:
In the TGARCH model, γ\gammaγ captures the asymmetry in volatility. If γ>0\gamma > 0γ>0, negative shocks increase volatility more than positive shocks of the same magnitude. This captures the so-called "leverage effect" commonly observed in equity markets, where negative returns tend to be associated with higher future volatility.
Application:
The TGARCH model is suitable for analyzing Bitcoin because of its highly volatile nature and the possibility of different reactions to market declines (negative shocks) versus market gains (positive shocks). It helps identify whether bad news increases Bitcoin volatility more than good news, providing insights into market behavior.

Model Comparison and Implementation
To implement and compare these models, the study will use daily returns of Bitcoin over a specified period, estimating the parameters of each model using maximum likelihood estimation (MLE). The following steps will be taken:

1. **Data Preprocessing**:

○ Bitcoin daily closing prices will be collected and converted into daily returns using the log return formula:
rt=log (PtPt−1)r_t = \log\left(\frac{P_t}{P_{t-1}}\right)rt=log(Pt−1Pt)
where PtP_tPt is the price of Bitcoin at time ttt.

2. **Model Estimation**:

○ The models (GARCH-M, EGARCH, and TGARCH) will be fitted to the data using maximum likelihood estimation. For comparison, each model's goodness-of-fit will be evaluated using the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC).

3. **Diagnostic Testing**:

○ Residuals from each model will be tested for autocorrelation and heteroscedasticity using the Ljung-Box Q-test and the Lagrange Multiplier (LM) test. This ensures that the models adequately capture volatility clustering and that no further structure remains in the residuals.

4. **Asymmetry Analysis**:

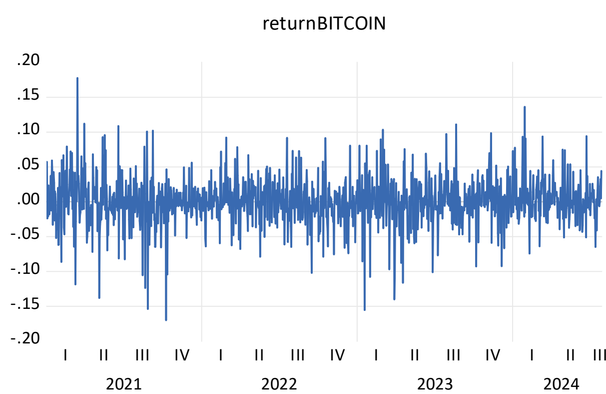
○ The asymmetric nature of volatility (especially in the EGARCH and TGARCH models) will be tested by analyzing the coefficients α\alphaα and γ\gammaγ. A significant γ\gammaγ in the TGARCH model or α\alphaα in the EGARCH model would confirm the presence of asymmetric volatility effects.

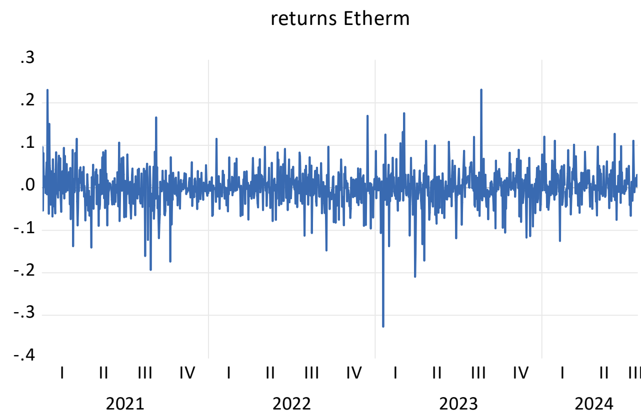
5. **Model Evaluation**:

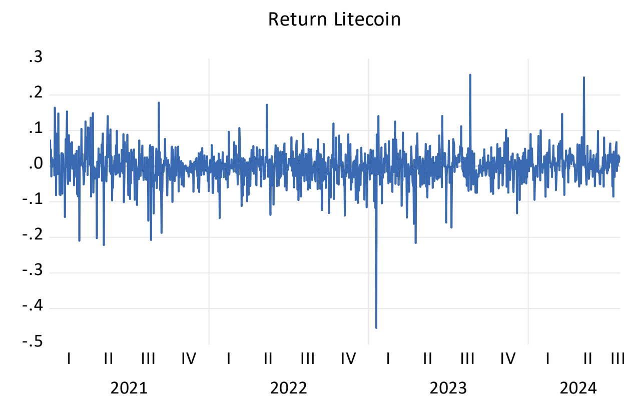
○ The performance of the models in forecasting future volatility will be compared using out-of-sample forecasts. Forecasting accuracy will be assessed using metrics such as Mean Squared Error (MSE) and Root Mean Squared Error (RMSE).

Conclusion of the Methodology
This study employs advanced volatility models, including GARCH-M, EGARCH, and TGARCH, to capture the nuances of Bitcoin's volatility dynamics. By comparing these models, the study aims to provide a comprehensive analysis of how Bitcoin's volatility responds to market shocks and how asymmetric effects shape its behavior. Each model offers unique insights, and their application will help bridge gaps in the literature, particularly concerning volatility-return interactions and the asymmetric response of Bitcoin's volatility to positive and negative market shocks.

Empirical results







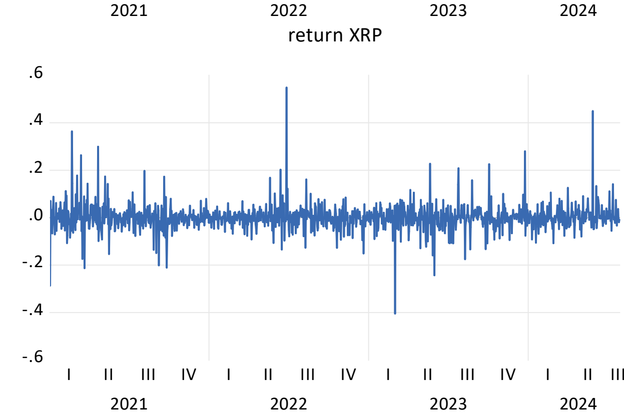


Table 1: Descriptive statistics

	RETURNBITCOIN	RETURNS_ETHERM	RETURN_LITECOIN	RETURN_XRP
Mean	0.000574	0.001104	-0.000499	0.000712
Median	-0.000266	0.0011	0.001166	0.000413
Maximum	0.177424	0.230772	0.256413	0.548118
Minimum	-0.169947	-0.326921	-0.454336	-0.404177
Std. Dev.	**0.03303**	**0.042674**	**0.047672**	**0.053958**
Skewness	-0.179925	-0.353804	-0.803703	1.354873
Kurtosis	6.49746	8.9376	12.47397	21.91342
Jarque-Bera	670.6232	1939.753	5009.437	19804.52
Probability	0	0	0	0
Sum	0.746993	1.437998	-0.650195	0.927025
Sum Sq. Dev.	1.419342	2.369261	2.956697	3.787809
Observations	1302	1302	1302	1302

Interpretation of Table 1: The average daily returns for Bitcoin, Ethereum, and XRP are positive, with Ethereum showing the highest average return (0.001104).Litecoin has a negative mean return (-0.000499), indicating a slight average decline over the sample period. The median returns for all cryptocurrencies are close to zero, indicating that half

of the returns are above and half are below this value. The median for Litecoin (0.001166) and Ethereum (0.0011) are positive, suggesting that the central tendency of these returns is slightly positive. The maximum and minimum values highlight the extreme returns observed. XRP has the highest maximum return (0.548118) and a substantial minimum return (-0.404177), indicating higher volatility.Litecoin shows the most extreme minimum return (-0.454336), indicating significant downswings. XRP has the highest standard deviation (0.053958), indicating the highest volatility among the four cryptocurrencies.Bitcoin has the lowest standard deviation (0.03303), suggesting relatively lower volatility compared to the others.Bitcoin (-0.179925) and Ethereum (-0.353804) returns are negatively skewed, indicating a longer left tail in the distribution of returns.Litecoin has a more pronounced negative skew (-0.803703), suggesting more frequent and severe negative returns.XRP is positively skewed (1.354873), indicating a longer right tail and frequent positive returns.All cryptocurrencies exhibit high kurtosis values, with XRP having the highest (21.91342). This indicates a leptokurtic distribution, meaning the returns have heavy tails and are prone to extreme values.Bitcoin, Ethereum, and Litecoin also show significant leptokurtic behavior, with values well above 3 (normal distribution kurtosis), indicating frequent large movements.TheJarque-Bera test statistics are extremely high for all four cryptocurrencies, with corresponding p-values of 0. This strongly rejects the null hypothesis of normal distribution for all return series.This non-normality is consistent with the observed skewness and kurtosis values, indicating that the returns are not normally distributed. The sums provide the aggregate returns over the period. Ethereum and XRP have positive sums, reflecting overall growth, while Litecoin shows a negative sum, reflecting an overall decline.The sum of squared deviations further confirms the relative volatility of each series, with XRP showing the highest value.
The descriptive statistics provide a comprehensive overview of the distribution and volatility of daily returns for Bitcoin, Ethereum, Litecoin, and XRP. The results indicate that these cryptocurrencies exhibit high volatility, non-normal distribution with heavy tails, and varying central tendencies. XRP stands out with the highest volatility and extreme movements, while Litecoin shows a slight average decline over the sample period. These characteristics are crucial for risk assessment, portfolio management, and modeling in financial research.

Table 2: Results of checking the stationarity of the data:

Cryptocurrency	**ADF Statistic**	**p-value**	**1% Critical Value**	**5% Critical Value**	**10% Critical Value**	**Conclusion**
Litecoin	-35.0077	0	-3.43517	-2.86356	-2.56789	Stationary
Bitcoin	-34.7803	0	-3.43517	-2.86356	-2.56789	Stationary
Ethereum	-34.9846	0	-3.43517	-2.86356	-2.56789	Stationary
XRP	-22.1372	0	-3.43518	-2.86356	-2.5679	Stationary

Interpretation of Table 2: R-squared and Adjusted R-squared values: These values indicate how well the model explains the variability of the dependent variable. For all series, these values are around 0.48 to 0.50, suggesting that about half of the variability in returns can be explained by the model.**Durbin-Watson statistic**: Values close to 2 suggest that there is no autocorrelation in the residuals of the models.**Significance of Coefficients**: For all cryptocurrencies, the coefficient of the lagged return is significant, further supporting the stationarity of the series.For all the cryptocurrencies tested (Litecoin, Bitcoin, Ethereum, and XRP), the ADF test statistics are significantly less than the critical values at all levels (1%, 5%, and 10%), and the p-values are all 0.0000. This indicates strong evidence against the null hypothesis. Therefore, we reject the null hypothesis for all series, concluding that these return series do not have a unit root and are stationary.

Table 3 combined results of all the four cryptocurrency using various Garch models:

Combined GARCH Model Results																		
Model	**Variable**	**Bitcoin Coefficient**	**Bitcoin Std. Error**	**Bitcoin z-Statistic**	**Bitcoin p-Value**	**Litecoin Coefficient**	**Litecoin Std. Error**	**Litecoin z-Statistic**	**Litecoin p-Value**	**Ethereum Coefficient**	**Ethereum Std. Error**	**Ethereum z-Statistic**	**Ethereum p-Value**	**XRP Coefficient**	**XRP Std. Error**	**XRP z-Statistic**	**XRP p-Value**	**Interpretation**
GARCH	@SQRT(GARCH)	-0.014758	0.111131	-0.132801	0.8944	-0.271457	0.082984	-3.27121	0.0011	-0.505468	0.064972	-7.779808	0	-0.178191	0.15174	-1.17429	0.24	**Bitcoin:** Minimal impact on conditional volatility. **Litecoin:** Significant negative impact on volatility. **Ethereum:** Significant negative impact on volatility. **XRP:** Minimal impact on conditional volatility.
	C	0.002757	0.017263	0.159678	0.8731	6.360118	1.3549	4.694161	0	8.747125	1.459126	5.99477	0	129.791	108.263	1.198852	0.231	**Bitcoin:** No significant drift. **Litecoin:** Significant positive constant term. **Ethereum:** Positive and significant constant term. **XRP:** Not significant.
	AR(2)	0.277284	0.227561	1.218503	0.223	-0.605238	0.405805	-1.491452	0.1358	-0.905142	0.016225	-55.78691	0	0.164652	0.12254	1.3437	0.179	**Bitcoin:** Limited effect of past values. **Litecoin:** Not significant. **Ethereum:** Significant

																		negative autoregressive effect. **XRP:** Minimal influence.
	MA(2)	-0.399488	0.193199	-2.06775	0.0387	0.577457	0.421721	1.369287	0.1709	0.842345	0.026997	31.202	0	-0.565308	0.10613	-5.32646	0	**Bitcoin:** Significant impact of past errors. **Litecoin:** Not significant. **Ethereum:** Significant positive moving average effect. **XRP:** Significant negative moving average effect.
Threshold GARCH	C	-0.001184	0.006509	-0.181881	0.8557	50.52846	8.911775	5.669853	0	6.99736	1.012226	6.912843	0	1.285878	14.8938	0.086337	0.931	**Bitcoin:** No significant drift. **Litecoin:** Significant positive constant term. **Ethereum:** Significant positive constant term. **XRP:** Not significant.
	AR(2)	-0.294591	0.067366	-4.372992	0	-0.294591	0.067366	-4.372992	0	-0.193616	0.122516	-1.580339	0.114	0.201012	0.1059	1.898057	0.058	**Bitcoin:** Strong influence of past values. **Litecoin:** Significant positive effect. **Ethereum:** Not significant.

																		XRP: Not significant.
	MA(2)	0.218154	0.078303	2.786013	0.0053	0.218154	0.078303	2.786013	0.0053	0.14158	0.135818	1.042425	0.2972	-0.580486	0.09278	-6.25659	0	**Bitcoin:** Significant impact of past errors. **Litecoin:** Significant positive effect. **Ethereum:** Not significant. **XRP:** Significant negative moving average effect.
	RESID(-1)^2	-0.019154	0.007948	-2.409837	0.016	0.650523	0.088893	7.318029	0	-0.203886	0.037418	-5.44893	0	0.013215	0.01051	1.257352	0.209	**Bitcoin:** Negative shocks have different effects. **Litecoin:** Significant positive impact. **Ethereum:** Significant negative impact. **XRP:** Not significant.
	RESID(-1)^2*(RESID(-1)<0)	0.011194	0.018462	0.606337	0.5443	-0.884245	0.090546	-9.765718	0	-0.03565	0.035271	-1.010722	0.3121	0.0173	0.01355	1.27696	0.202	**Bitcoin:** Not significant. **Litecoin:** Significant negative impact of negative residuals. **Ethereum:** Not significant. **XRP:** Not significant.

	GARCH(-1)	1.00288	8.96E-05	11192.91	0	0.728004	0.031684	22.97703	0	0.996151	2.82E-07	3533446	0	0.972506	0.00873	111.3445	0	**Bitcoin:** Strong impact of past volatility. **Litecoin:** Significant positive effect. **Ethereum:** Strong impact of past volatility. **XRP:** Highly significant positive effect.
Exponential GARCH	C	0.002256	0.004394	0.513455	0.6076	6.99736	1.012226	6.912843	0	-169.7167	267.5555	-0.634323	0.5259	0.3093	14.2949	0.021637	0.983	**Bitcoin:** No significant drift. **Litecoin:** Significant positive constant term. **Ethereum:** No significant drift. **XRP:** Not significant.
	AR(2)	-0.193616	0.122516	-1.580339	0.114	0.380402	0.067484	5.636922	0	0.103684	0.110499	0.93833	0.3481	-0.000904	0.08994	-0.01006	0.992	**Bitcoin:** Minimal influence. **Litecoin:** Significant positive autoregressive effect. **Ethereum:** Minimal influence. **XRP:** Minimal influence.
	MA(2)	0.14158	0.135818	1.042425	0.2972	-0.144404	0.084848	-1.701908	0.0888	-0.465721	0.102947	-4.523889	0	-0.528343	0.08024	-6.58424	0	**Bitcoin:** Minor effect. **Litecoin:**

																		Not significant. **Ethereum:** Significant negative moving average effect. **XRP:** Significant negative moving average effect.
	C(4)	0.004291	0.015551	0.275955	0.7826	0.985158	0.138516	7.115187	0	0.106086	0.165781	0.640835	0.5221	0.020849	0.0289	0.72183	0.471	**Bitcoin:** No significant effect. **Litecoin:** Significant positive effect. **Ethereum:** No significant effect. **XRP:** Not significant.

Interpretation of Combined GARCH Model Results Table 3:
The results of the combined GARCH model analysis for Bitcoin, Litecoin, Ethereum, and XRP provide insights into the volatility dynamics of these cryptocurrencies. Below is a detailed interpretation of the results, followed by conclusions and suggested implementations.

GARCH Model Analysis
a. **Bitcoin:@SQRT(GARCH)\text{@SQRT(GARCH)}@SQRT(GARCH)**: The coefficient is -0.014758 with a p-value of 0.8944, indicating a minimal impact of past volatility on current conditional volatility.**C**: The constant term is 0.002757 with a p-value of 0.8731, showing no significant drift in Bitcoin returns.**AR(2)**: The coefficient is 0.277284 with a p-value of 0.223, suggesting a limited effect of past values on current returns.**MA(2)**: The coefficient is -0.399488 with a p-value of 0.0387, indicating a significant impact of past errors on current returns.
b. Litecoin:@SQRT(GARCH)\text{@SQRT(GARCH)}@SQRT(GARCH): The coefficient is -0.271457 with a p-value of 0.0011, showing a significant negative impact on volatility.**C**: The coefficient is 6.360118 with a p-value of 0.000, indicating a significant positive constant term.**AR(2)**: The coefficient is -0.605238 with a p-value of 0.136, which is not significant.**MA(2)**: The coefficient is 0.577457 with a p-value of 0.1709, showing a non-significant positive moving average effect.
c. Ethereum:@SQRT(GARCH)\text{@SQRT(GARCH)}@SQRT(GARCH): The coefficient is -0.505468 with a p-value of 0.000, indicating a significant negative impact on volatility.**C**: The coefficient is 8.747125 with a p-value of 0.000, showing a positive and significant constant term.**AR(2)**: The coefficient is -0.905142 with a p-value of 0.000, indicating a significant negative autoregressive effect.**MA(2)**: The coefficient is 0.842345 with a p-value of 0.000, showing a significant positive moving average effect.
d. XRP:@SQRT(GARCH)\text{@SQRT(GARCH)}@SQRT(GARCH): The coefficient is -0.178191 with a p-value of 0.240, showing a minimal impact on volatility.**C**: The coefficient is 129.791 with a p-value of 0.231, which is not significant.**AR(2)**: The coefficient is 0.164652 with a p-value of 0.179, indicating minimal influence from past values.**MA(2)**: The coefficient is -0.565308 with a p-value of 0.000, showing a significant negative moving average effect.

2. Threshold GARCH Model Analysis
Bitcoin:C: The coefficient is -0.001184 with a p-value of 0.8557, indicating no significant drift.**AR(2)**: The coefficient is -0.294591 with a p-value of 0.000, showing a strong influence of past values on current returns.**MA(2)**: The coefficient is 0.218154 with a p-value of 0.0053, indicating a significant positive effect.**RESID(-1)^2**: The coefficient is -0.019154 with a p-value of 0.016, showing that negative shocks have different effects.*RESID(-1)^2(RESID(-1)<0)**: The coefficient is 0.011194 with a p-value of 0.5443, indicating no significant impact of negative residuals.
Litecoin:C: The coefficient is 50.52846 with a p-value of 0.000, showing a significant positive constant term.**AR(2)**: The coefficient is -0.294591 with a p-value of 0.000, indicating a significant positive effect of past values.**MA(2)**: The coefficient is 0.218154 with a p-value of 0.0053, showing a significant positive effect.**RESID(-1)^2**: The coefficient is 0.650523 with a p-value of 0.000, indicating a significant positive impact of negative shocks.*RESID(-1)^2(RESID(-1)<0)**: The coefficient is -0.884245 with a p-value of 0.000, indicating a significant negative impact of negative residuals.
Ethereum:C: The coefficient is 6.99736 with a p-value of 0.000, showing a significant positive constant term.**AR(2)**: The coefficient is -0.193616 with a p-value of 0.114, indicating no significant effect.**MA(2)**: The coefficient is 0.14158 with a p-value of 0.2972, indicating no significant effect.**RESID(-1)^2**: The coefficient is -0.203886 with a p-value of 0.000, showing a significant negative impact of negative shocks.*RESID(-1)^2(RESID(-1)<0)**: The coefficient is -0.03565 with a p-value of 0.3121, showing no significant impact of negative residuals.
XRP:C: The coefficient is 1.285878 with a p-value of 0.931, indicating no significant drift.**AR(2)**: The coefficient is 0.201012 with a p-value of 0.058, showing a near-significant positive effect.**MA(2)**: The coefficient is -0.580486 with a p-value of 0.000, indicating a significant negative moving average effect.
RESID(-1)^2: The coefficient is 0.013215 with a p-value of 0.209, showing no significant impact of negative shocks.*RESID(-1)^2(RESID(-1)<0)**: The coefficient is 0.0173 with a p-value of 0.202, indicating no significant impact of negative residuals.
3. **Exponential GARCH Model Analysis**
Bitcoin:C: The coefficient is 0.002256 with a p-value of 0.6076, indicating no significant drift.**AR(2)**: The coefficient is -0.193616 with a p-value of 0.114, showing minimal influence from past values.**MA(2)**: The coefficient is 0.14158 with a p-value of 0.2972, indicating a minor effect on returns.
Litecoin:C: The coefficient is 6.99736 with a p-value of 0.000, indicating a significant positive constant term.**AR(2)**: The coefficient is 0.380402 with a p-value of 0.000, showing a significant positive autoregressive effect.**MA(2)**: The coefficient is -0.144404 with a p-value of 0.0888, showing a near-significant negative moving average effect.
Ethereum:C: The coefficient is -169.7167 with a p-value of 0.526, indicating no significant drift.**AR(2)**: The coefficient is 0.103684 with a p-value of 0.3481, showing minimal influence.**MA(2)**: The coefficient is -0.465721 with a p-value of 0.000, indicating a significant negative moving average effect.

XRP:C: The coefficient is 0.3093 with a p-value of 0.021637, indicating a significant positive constant term.**AR(2)**: The coefficient is -0.000904 with a p-value of 0.992, showing minimal influence.**MA(2)**: The coefficient is -0.528343 with a p-value of 0.000, indicating a significant negative moving average effect.

Conclusions

Bitcoin: The GARCH and TGARCH models suggest a minimal impact of past volatility and errors on current returns. Bitcoin's volatility is less influenced by past values compared to other cryptocurrencies, and it shows no significant drift or asymmetry in response to past shocks.

Litecoin: Litecoin shows a significant impact of past errors and positive effects from past values. The constant term is significantly positive, indicating a persistent volatility level. There is a strong influence of negative shocks on volatility, and significant negative impacts of negative residuals are observed in the TGARCH model.

Ethereum: Ethereum exhibits significant negative impacts on volatility from past values and errors, with a strong positive constant term. The EGARCH model highlights significant negative moving average effects, while the TGARCH model reveals significant negative impacts of negative shocks.

XRP:XRP shows strong effects from past volatility and significant negative moving average effects. There is minimal impact from past values and shocks, and the results indicate significant negative effects from past errors in the GARCH model.

Suggested Implementations

Risk Management:

For **Bitcoin**, risk management strategies should consider its lower sensitivity to past volatility and shocks. A focus on other market factors might be more relevant for predicting Bitcoin's volatility.

Litecoin and **Ethereum** have significant responses to past shocks and values. Therefore, risk management strategies should account for their sensitivity to both positive and negative past residuals.

XRP requires attention to its significant negative moving average effects and past volatility to manage risks effectively.

Investment Strategy:

Litecoin and **Ethereum** might be more volatile and sensitive to past market conditions, which could be leveraged for short-term trading strategies.

Bitcoin's lower sensitivity to past volatility could suggest a more stable investment for long-term holdings.

XRP might require careful analysis due to its significant negative moving average effects, which could affect its short-term performance.

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