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A Study on Existence of Fixed Points in Intuitionistic Fuzzy Strong b-Metric Spaces

Kshama Pandey[1*] , Pranjali Sharma[2], Shailesh Dhar Diwan[3]

[1*]Department of Mathematics, Govt. Engineering College, Raipur(CG), kshamapandey11@gmail.com
[2]Department of Mathematics, SSIPMT, Raipur(CG)
[3]Department of Mathematics, Govt. Engineering College, Raipur(CG)

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Abstract

This research paper establishes the existence of fixed points for mappings in intuitionistic fuzzy strong b-metric spaces, advancing the understanding of these mathematical structures. We introduce some theorems that provide sufficient conditions for the existence of unique fixed points. These theorems introduce approaches to ensuring the existence and uniqueness of fixed points for mappings in intuitionistic fuzzy strong b-metric spaces under specific contractive conditions. Illustrative examples provided to support our findings. These findings contribute to the advancement of fixed point theory in intuitionistic fuzzy strong b-metric spaces, offering new insights and potential applications in mathematical modeling and optimization problems. The results provide valuable insights for researchers and mathematicians working in related areas.

Keywords: fixed point, fuzzy metric space, intuitionistic fuzzy metric space, intuitionistic fuzzy strong b-metric space.

Subject Classification: 47H10, 54H25.

1. Introduction:

In the realm of pure mathematics, fixed-point theory stands as a vibrant and dynamic area of research, profoundly impacting nonlinear analysis. Scholars have extensively explored this field, yielding significant contributions [1–5]. The groundbreaking concept of fuzzy set theory, introduced by Zadeh [6], has enabled the description and manipulation of vagueness and imprecision. Intuitionistic fuzzy set theory, a generalization of fuzzy sets, offers even greater versatility.

In 1975, Kramosil and Michálek [7] pioneered fuzzy metric spaces, extending probabilistic metric spaces. Later, George and Veeramani [8] refined this notion. Grabiec [9] explored completeness in fuzzy metric spaces, while Fang [17] established fixed-point theorems for contractive-type mappings. Beyond fuzzy metric spaces, Bakhtin's [18] b-metric spaces relaxed the triangle inequality, leading to Czerwik's [19] research.

Recently, Kirk and Shahzad [21] introduced strong b-metric spaces, ensuring open balls are open. Oner and Sostak [20] subsequently defined strong fuzzy b-metric spaces, combining the benefits of metric and fuzzy metric spaces. Intuitionistic fuzzy strong b-metric spaces, our focus, inherit these advantages.

Intuitionistic fuzzy metric spaces, introduced by Park [22], extend fuzzy metric spaces by incorporating intuitionistic fuzzy sets. These spaces possess unique properties, such as the separation of membership and non-membership functions. Atanassov [23] explored intuitionistic fuzzy sets, highlighting their potential in modeling uncertain systems. The study of intuitionistic fuzzy metric spaces has since gained momentum, with researchers investigating fixed-point theory [36, 37] and topological properties [38].

Notably, the work of Saadati et al. [39] on intuitionistic fuzzy metric spaces provides a foundation for our research. Their results on completeness and convergence in these spaces inform our investigation. Furthermore, the fixed-point theorems established by Sedghi et al. [40] for intuitionistic fuzzy metric spaces serve as a precursor to our findings.

Notably, Shazia Kanwal [20] investigated the existence of fixed points in fuzzy strong b-metric spaces, providing valuable insights. Building upon Kanwal's work, we extend the research to intuitionistic fuzzy strong b-metric spaces.

This paper aims to further investigate the existence of fixed points in intuitionistic fuzzy strong b-metric spaces. We establish fixed-point theorems, providing sufficient conditions for unique fixed points. Illustrative examples demonstrate the applicability of our results. our findings, contribute to the advancement of fixed-point theory in intuitionistic fuzzy mathematical structures, offering new insights and potential applications in mathematical modeling and optimization problems.

2. Preliminaries

This section lays the groundwork for our main findings by introducing fundamental definitions and concepts. Drawing from established research, we outline key notions that provide a crucial framework for grasping the contributions of this study. By reviewing relevant theoretical foundations, we establish a solid basis for understanding the subsequent results.

"Definitions 1 ([3]): A mapping $f: [0,1] \times [0,1] \to [0,1]$ is called continuous t-norm if it satisfies the following axioms:

1. Symmetry: f(x, y) = f(x, y) , $\forall x, y \in [0,1]$
2. Monotonicity: $f(x_1, y_1) \leq f(x_2, y_2)$, if $x_1 \leq x_2$, $y_1 \leq y_2$
3. Associativity: f(f(x, y),z) = f(x, f(y, z))
4. Boundary Condition: f(1, a) = a, $\forall a \in [0,1]$

"

"Definition 2 ([41]): A binary operation $\lozenge: [0,1] \times [0,1] \to [0,1]$ is called a continuous t-conorm if ◊ satisfies the following:

1. $a \lozenge 0 = 0, \quad \forall a \in [0,1]$
2. $a \lozenge b = b \lozenge a \ and \ a \lozenge (b \lozenge c) = (a \lozenge b) \lozenge c \quad \forall a, b, c \in [0,1]$
3. If $a \leq c, b \leq d$ then $a \lozenge b \leq c \lozenge d$, $\forall a, b, c, d \in [0,1]$
4. ◊ is continuous."

"Definition 3 ([7]): Triple (X, M, *) is called fuzzy metric spaces, where X is an arbitrary non-empty set, * is t-norm and M is fuzzy set defined on X × X ×[0,∞) such that for all x, y, z ∈X the following axioms satisfied

(M1) M(x, y, 0) = 0

(M2) M(x, y, t) = 1, iff x= y, t≥0

(M3) M(x, y, t) = M(y, x, t)

(M4) M(x, z, t + s) ≥ M(x, y, t) * M(y, z, s)

(M5) M(x, y, .) : $[0, \infty) \to [0,1]$ is left continuous."

"Definition 4([22]): Intuitionistic fuzzy metric space defined as 5 tuple (X, M, N, *, ◊), where X is an arbitrary non-empty set, * is a binary operation represents t-norm, and ◊ is a binary operation represents continuous t-conorm, M and N are fuzzy set defined on X × X ×[0,∞) such that for all x, y, z ∈X the following axioms satisfied

(IFM1) M (x, y, 0) = 0

(IFM2) M (x, y, t) = 1, iff x = y, t≥0

(IFM3) M (x, y, t) = M (y, x, t) , t≥0

(IFM4) M (x, z, t + s) ≥ M (x, y, t) * M (y, z, s), t≥0, s≥0

(IFM5) M (x, y, .) : $[0, \infty) \to [0,1]$ is left continuous.

(IFM6) M (x, y, t) = 1 as t→∞

(IFM7) N (x, y, 0) = 1

(IFM8) N (x, y, t) = 0, iff x = y, t≥0

(IFM9) N (x, y, t) = N (y, x, t), t≥0

(IFM10) N (x, z, t + s) ≤ N (x, y, t) ◊ N (y, z, s), t≥0, s≥0
(IFM11) N (x, y, .) : $[0, \infty) \to [0,1]$ is right continuous.
(IFM12) N (x, y, t) = 0 as t→∞
(IFM13) M (x, z, t) + N (x, y, t) ≤ 1, t≥0"

"Definition 5([42]): A quadruple (X, M, *, b) is called fuzzy b-metric space, where X is an arbitrary non-empty set, * is t-norm and M is a fuzzy set defined on X × X ×[0,∞) if there exists b ≥ 1 such that for all x, y, z ∈X the following axioms satisfied
(b1) M(x, y, 0) = 0
(b2) M(x, y, t) = 1, iff x = y, t≥0
(b3) M(x, y, t) = M(y, x, t) , t≥0
(b4) M(x, z, b.(t + s)) ≥ M(x, y, t) * M(y, z, s)
(b5) M(x, y, .) : $[0, \infty) \to [0,1]$ is left continuous."

"Definition 6([20]): A quadruple (X, M, *, b) is called fuzzy Strong b-metric space, where X is an arbitrary non-empty set, $b \geq 1$ be a real no, * is t-norm and M is a fuzzy set defined on X × X ×[0,∞) if x, y, z ∈X the following axioms satisfied
(sbM1) M(x, y, 0) = 0
(sbM2) M(x, y, t) = 1, iff x = y, t≥0
(sbM3) M(x, y, t) = M(y, x, t) , t≥0
(sbM4) M(x, z, t + b. s) ≥ M(x, y, t) * M(y, z, s)
(sbM5) M(x, y, .) : $[0, \infty) \to [0,1]$ is left continuous."

Remark 6.1[(30)]:
"Let (X, M, *, b) be a Non-Archimedean Fuzzy Strong b-Metric Space:
1. A sequence $\{b_n\}$ in X is said to be convergent and converges to a point $b \in X$ if
$$\lim_{n\to\infty} M(b_n, b, k) = 1, \forall t \geq 0.$$
2. A sequence $\{a_n\}$ in X is said to be a Cauchy sequence if for any $0 < \varepsilon < 1$ and $u > 0$, $\exists n_0 \in N$ such that $M(a_n, a_m, t) < \varepsilon$, $\forall n, m \geq n_0$.
3. A Fuzzy Strong b- metric space is called complete if every Cauchy sequence in it is convergent."

"Definition 7: Intuitionistic fuzzy strong b metric space defined as (X, M, N, *, ◊, b), where X is an arbitrary non-empty set, * is a binary operation represents t-norm, and ◊ is a binary operation represents continuous t-conorm, M and N are fuzzy set defined on X × X ×[0,∞) such that for all x, y, z ∈X the following axioms satisfied:

(IFsbM1) M(x, y, 0) = 0
(IFsbM2) M (x, y, t) = 1, iff x = y, t≥0
(IFsbM3) M (x, y, t) = M (y, x, t) , t≥0
(IFsbM4) M (x, z, (t + b.s)) ≥ M (x, y, t) * M (y, z, s), t≥0, s≥0
(IFsbM5) M (x, y, .) : $[0, \infty) \to [0,1]$ is left continuous.
(IFsbM6) M (x, y, t) = 1 as t→∞
(IFsbM7) N (x, y, 0) = 1
(IFsbM8) N (x, y, t) = 0, iff x = y, t≥0
(IFsbM9) N (x, y, t) = N (y, x, t), t≥0
(IFsbM10) N (x, z, t + b.s) ≤ N (x, y, t) ◊ N (y, z, s), t≥0, s≥0
(IFsbM11) N (x, y, .) : $[0, \infty) \to [0,1]$ is right continuous.
(IFsbM12) N (x, y, t) = 0 as t→∞
(IFsbM13) M (x, z, t) + N (x, y, t) ≤ 1, t≥0."

"Definition 7.1: Consider $(I, P, Q, *, \lozenge, b)$ be an intuitionistic fuzzy strong b-metric space. A sequence $\{q_n\}$ in I converges to $q \in I$, if for every $s > 0$
$$\lim_{n\to\infty} P(q_n, q, k) = 1$$

$$\lim_{n\to\infty} Q(q_n, q, k) = 0$$

Definition 7.2: Consider $(I, P, Q,*,\lozenge, b)$ be an intuitionistic fuzzy strong b-metric space. A sequence $\{q_n\}$ in I is Cauchy if, for any 0 < ε < 1 and u > 0, there exists $n_0 \in \mathbb{N}$ such that:

$$P(q_n, q_m, u) > 1 - \varepsilon$$
$$Q(q_n, q_m, u) < \varepsilon$$

for all n, m ≥ n_0.

Definition 7.3: The space $(I, P, Q,*,\lozenge, b)$ is complete if every Cauchy sequence in I converges to a point within I."

"Definition 8 ([43]): Let (X, d) be a metric space. A mapping f: X ⟶ X is known as Banach contraction on f if there is a positive real number 0 < α < 1 such that ∀x, y ∈ X:
d(fx, fy) ≤ αd(x, y)."

"Definition9 ([29]): Let (X, d) be a metric space and f: X⟶ X be a mapping if ∃ α ∈ (0, 1/2) such that, for all x_1, x_2 ∈ X, we have
d $(fx_1, fx_2) \leq \alpha\{d(x_1, fx_1) + d(x_2, fx_2)\}$
Then, f is known as Kannan contraction."

"Definition10 ([35]): Let (X, d) be a metric space and $G: X \to X$ be a mapping if there exists α∈(0,1/2) such that, for all $a_1, a_2 \in X$, we have $d(Ga_1, Ga_2) \leq \alpha\{d(a_1, Ga_2) + d(a_2, Ga_1)\}$
Then, G is known as Chatterjee contraction."

3. Main Result

Significant results ensure the existence and uniqueness of fixed points in intuitionistic fuzzy strong b-metric spaces. Examples are also provided to illustrate the strength of these results.

Theorem 1

Let $(I, P, Q,*,\lozenge, b)$ be a complete Intuitionistic Fuzzy Strong b metric space, * and ◊ are continuous t-norm and t-conorm respectively and $0 < k < 1$. $P(l, m, s)$ are strictly increasing and $Q(l, m, s)$ are strictly decreasing in a third variable s

$$\lim_{s\to\infty} P(l, m, s) = 1 \qquad \text{\_\_\_\_\_\_\_\_(1)}$$
$$\lim_{s\to\infty} Q(l, m, s) = 0 \qquad \text{\_\_\_\_\_\_\_\_(2)}$$

Consider a mapping $h: I \to I$ satisfies

$$P(hl, hm, ks) \geq P(l, m, s) \quad \forall l, m \in I \;\&\; s \geq 0$$
$$Q(hl, hm, ks) \leq Q(l, m, s) \quad \forall l, m \in I \;\&\; s \geq 0$$

Then, a unique fixed point of h exists in I.

Proof: let $\{q_n\}$ be any sequence in I and $q_0 \in I$ be any arbitrary element, So that

$$q_n = hq_n$$
$$= h^n q_0 \qquad (n \in N)$$

$$P(q_n, q_{n+1}, ks) = P(h^n q_0, h^{n+1} q_0, ks)$$
$$\geq P(h^{n-1} q, h^n q_0, s) \quad \{\because P(l, m, s) is\ strictly\ increasing\}$$
$$= P(q_{n-1}, q_n, s)$$
$$= P(h^{n-1} q_0, h^n q_0, s)$$
$$\geq P\left(h^{n-2} q_0, h^{n-1} q_0, \frac{s}{k}\right) \qquad \text{\_\_\_\_\_\_(3)}$$

$$= P\left(q_{n-2}, q_{n-1}, \frac{s}{k}\right)$$

$$\geq P\left(q_0, q_1, \frac{s}{k^{n-1}}\right)$$

So, $P(q_n, q_{n+1}, bs) \geq P(q_0, q_1, \frac{s}{k^{n-1}})$

$$Q(q_n, q_{n+1}, bs) = Q(h^n q_0, h^{n+1} q_0, bs)$$

$$\leq Q(h^{n-1} q, h^n q_0, s) \quad \{\therefore Q(l, m, s) is\ strictly\ decreasing\}$$
$$= Q(q_{n-1}, q_n, s)$$
$$= Q(h^{n-1} q_0, h^n q_0, s)$$
$$\leq Q\left(h^{n-2} q_0, h^{n-1} q_0, \frac{s}{k}\right) \quad \text{\_\_\_\_\_\_\_\_\_(4)}$$
$$= Q\left(q_{n-2}, q_{n-1}, \frac{s}{k}\right)$$
$$\leq Q\left(q_0, q_1, \frac{s}{k^{n-1}}\right)$$

$$Q(q_n, q_{n+1}, bs) \leq Q(q_0, q_1, \frac{s}{k^{n-1}})$$

By using (IFsbM4) and let for every $n \in N$ and positive integer r and $s \geq 0$

$$P(q_n, q_{n+r}, s) \geq P\left(q_n, q_{n+1}, \frac{s}{2}\right) * P(q_{n+1}, q_{n+r}, \frac{s}{2b})$$

$$\geq P\left(q_n, q_{n+1}, , \frac{s}{2}\right) * P\left(q_{n+1}, q_{n+2}, \frac{s}{4b}\right) * P(q_{n+2}, q_{n+r}, \frac{s}{4b^2})$$

$$\geq P\left(q_n, q_{n+1}, \frac{s}{2}\right) * P\left(q_{n+1}, q_{n+2}, \frac{s}{4b}\right) * P\left(q_{n+2}, q_{n+3}, \frac{s}{8b^2}\right) * P(q_{n+3}, q_{n+r}, \frac{u}{8b^3})$$

$$\geq P\left(q_n, q_{n+1}, \frac{s}{2}\right) * P\left(q_{n+1}, q_{n+2}, \frac{s}{4b}\right) * P\left(q_{n+2}, q_{n+3}, \frac{s}{4b^2}\right) * P\left(q_{n+3}, q_{n+4}, \frac{u}{8b^2}\right) * P\left(q_{n+4}, q_{n+r}, \frac{u}{8b^3}\right)$$

$$P(q_n, q_{n+r}, s) \geq P\left(q_n, q_{n+1}, \frac{s}{2}\right) * P\left(q_{n+1}, q_{n+2}, \frac{s}{4b}\right) * P\left(q_{n+2}, q_{n+3}, \frac{s}{8b^2}\right) * P\left(q_{n+3}, q_{n+4}, \frac{u}{8b^3}\right)$$
$$* P\left(q_{n+4}, q_{n+5}, \frac{u}{8b^2}\right) * P\left(q_{n+4}, q_{n+r}, \frac{u}{8b^3}\right) * - - -$$
$$* P\left(q_{n+r-1}, q_{n+r}, \frac{u}{2^{r-1} b^{r-1}}\right) \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_(4)}$$

Using (3) we have

$$P(q_n, q_{n+r}, s) \geq P\left(q_0, q_1, \frac{s}{2k^n}\right) * P\left(q_0, q_1, \frac{s}{2^2 k^{n+1}}\right) * - - -$$
$$* P\left(q_0, q_1, \frac{u}{2^{r-1} b^{r-1} k^{n+r-1}}\right) \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_(5)}$$

As $n \to \infty$, $k^n \to 0 \Rightarrow \frac{s}{2k^n} \longrightarrow \infty$. By using (1)

$$P(q_n, q_{n+r}, s) \geq 1 * 1 * 1 * - - - * 1 \qquad (r\ times)$$

$$P(q_n, q_{n+r}, s) \geq 1 \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_(6)}$$

By using (IFsbM10) and let for every $n \in N$ and positive integer r and $s \geq 0$

$$Q(q_n, q_{n+r}, s) \leq Q\left(q_n, q_{n+1}, \frac{s}{2}\right) \lozenge Q(q_{n+1}, q_{n+r}, \frac{s}{2b})$$

$$\leq Q\left(q_n, q_{n+1}, , \frac{s}{2}\right) \lozenge Q\left(q_{n+1}, q_{n+2}, \frac{s}{4b}\right) \lozenge Q(q_{n+2}, q_{n+r}, \frac{s}{4b^2})$$

$$\leq Q\left(q_n, q_{n+1}, \frac{s}{2}\right) \lozenge Q\left(q_{n+1}, q_{n+2}, \frac{s}{4b}\right) \lozenge Q\left(q_{n+2}, q_{n+3}, \frac{s}{8b^2}\right) \lozenge Q(q_{n+3}, q_{n+r}, \frac{u}{8b^3})$$

$$\leq Q\left(q_n, q_{n+1}, \frac{s}{2}\right) \lozenge Q\left(q_{n+1}, q_{n+2}, \frac{s}{4b}\right) \lozenge Q\left(q_{n+2}, q_{n+3}, \frac{s}{4b^2}\right) \lozenge Q\left(q_{n+3}, q_{n+4}, \frac{u}{8b^2}\right) \lozenge Q\left(q_{n+4}, q_{n+r}, \frac{u}{8b^3}\right)$$

$$Q(q_n, q_{n+r}, s) \leq Q\left(q_n, q_{n+1}, \frac{s}{2}\right) \lozenge Q\left(q_{n+1}, q_{n+2}, \frac{s}{4b}\right) \lozenge Q\left(q_{n+2}, q_{n+3}, \frac{s}{8b^2}\right) \lozenge Q\left(q_{n+3}, q_{n+4}, \frac{u}{8b^3}\right)$$
$$\lozenge Q\left(q_{n+4}, q_{n+5}, \frac{u}{8b^2}\right) \lozenge Q\left(q_{n+4}, q_{n+r}, \frac{u}{8b^3}\right) \lozenge - - -$$
$$\lozenge Q\left(q_{n+r-1}, q_{n+r}, \frac{u}{2^{r-1}b^{r-1}}\right) \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_(7)}$$

Using (4), we have

$$Q(q_n, q_{n+r}, s) \leq Q\left(q_0, q_1, \frac{s}{2k^n}\right) \lozenge Q\left(q_0, q_1, \frac{s}{2^2k^{n+1}}\right) \lozenge - - -$$
$$\lozenge Q\left(q_0, q_1, \frac{u}{2^{r-1}b^{r-1}k^{n+r-1}}\right) \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_(8)}$$

As $n \to \infty$, $k^n \to 0 \Rightarrow \frac{s}{2k^n} \longrightarrow \infty$. By using (2)

$$Q(q_n, q_{n+r}, s) \leq 0 \lozenge 0 \lozenge 0 \lozenge - - - \lozenge 0$$

$$Q(q_n, q_{n+r}, s) \leq 0 \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_(9)}$$

Equations (6) and (9) imply that $\{q_n\}$ is a Cauchy sequence and X is complete, So there exists a point p in X such that

$$\lim_{n\to\infty} q_n = p$$

By using (IFsbM4)

$$P(p, hp, s) \geq P\left(p, q_{n+1}, \frac{s}{2}\right) * P(q_{n+1}, hp, \frac{s}{2b})$$

$$\geq P\left(p, q_{n+1}, \frac{s}{2}\right) * P(hq_n, hp, \frac{s}{2b})$$

$$P(p, hp, s) \geq P\left(p, q_{n+1}, \frac{s}{2}\right) * P(q_n, p, \frac{s}{2bk})$$

When limit $n \to \infty$

$$P(p, hp, s) \geq P\left(p, p, \frac{s}{2}\right) * P(p, p, \frac{s}{2bk})$$
$$P(p, hp, s) \geq 1 * 1$$

$$P(p, hp, s) \geq 1$$

Hence, $$hp = p$$

By using (IFsbM10)

$$Q(p, hp, s) \leq Q\left(p, q_{n+1}, \frac{s}{2}\right) \lozenge Q(q_{n+1}, hp, \frac{s}{2b})$$

$$\leq Q\left(p, q_{n+1}, \frac{s}{2}\right) \lozenge Q(hq_n, hp, \frac{s}{2b})$$

$$Q(p, hp, s) \leq Q\left(p, q_{n+1}, \frac{s}{2}\right) \lozenge Q(q_n, p, \frac{s}{2bk})$$

When limit $n \to \infty$

$$Q(p, hp, s) \leq Q\left(p, p, \frac{s}{2}\right) \lozenge Q(p, p, \frac{s}{2bk})$$
$$Q(p, hp, s) \leq 1 * 1$$

$$Q(p, hp, s) \leq 0$$

Hence, $hp = p$

p is a fixed point of function h.

Uniqueness: Let p, p* be two fixed points of mapping h, then

$$hp = p \quad and \quad hp^* = p^*$$

$$P(hp, p, s) = 1 \; and \; P(hp^*, p^*, s) = 1$$
$$P(p, p^*, s) = P(hp, hp^*, s)$$
$$P(p, p^*, s) \geq P(p, p^*, \frac{s}{k})$$

This is a contradiction to the condition $P(l, m, s)$ is strictly increasing in variable s.

So, $p = p^*$

Now, $Q(hp, p, s) = 0 \; and \; Q(hp^*, p^*, s) = 0$
$$Q(p, p^*, s) = Q(hp, hp^*, s)$$
$$Q(p, p^*, s) \leq Q(p, p^*, \frac{s}{k})$$

This is a contradiction to the condition $Q(l, m, s)$ is strictly decreasing in variable s.

So, $p = p^*$
The fixed point is unique.

Example:1

Let $I = [0,1]$ and $P: I \times I \times [0, \infty) \to [0,1]$ & $Q: I \times I \times [0, \infty) \to [0,1]$ be defined as

$$P(l, m, s) = \begin{cases} \min\{l, m\} + \frac{s}{1+s}, & if \; s > 0 \\ \min\{l, m\} & if s = 0 \end{cases}$$

$$Q(l, m, s) = \begin{cases} \max\{l, m\} - \frac{s}{1+s} & if s > 0 \\ \max\{l, m\} & if s = 0 \end{cases}$$

If $h: I \to I$ be defined by $h(x) = kx$, where $k = \frac{3}{5}$. * is t-norm and $\lozenge$ is t-conorm.

When $s > 0$

$$P(hl, hm, ks) = P(kl, km, ks)$$

$$= \min\{kl, km\} + \frac{ks}{1+ks}$$

$$= \min\left\{\frac{3l}{5}, \frac{3m}{5}\right\} + \frac{\frac{3s}{5}}{1+\frac{3s}{5}}$$

$$\geq \min\{l, m\} + \frac{s}{1+s} = P(l, m, s)$$

When $s = 0$

$$P(hl, hm, 0) = P(kl, km, 0)$$

$$= \min\{kl, km\}$$

$$= \min\left\{\frac{3l}{5}, \frac{3m}{5}\right\}$$

$$\geq \min\{l, m\} = P(l, m, 0)$$

This implies $P(hl, hm, ks) \geq P(l, m, s)$

Similarly, when $s > 0$

$$Q(hl, hm, ks) = Q(kl, km, ks)$$

$$= \max\{kl, km\} - \frac{ks}{1+ks}$$

$$= \max\left\{\frac{3l}{5}, \frac{3m}{5}\right\} - \frac{\frac{3s}{5}}{1+\frac{3s}{5}}$$

$$\leq \max\{l, m\} - \frac{s}{1+s} = Q(l, m, s)$$

When $s = 0$

$$Q(hl, hm, 0) = Q(kl, km, 0)$$

$$= \max\{kl, km\}$$

$$= \max\left\{\frac{3l}{5}, \frac{3m}{5}\right\}$$

$$\leq \max\{l, m\} = Q(l, m, 0)$$

this implies $Q(hl, hm, ks) \leq Q(l, m, s)$

So, h contains a unique fixed point.

Corollary 1:
Let $(I, P, Q, *, \lozenge, b)$ be a complete Intuitionistic Fuzzy Strong b metric space, where * and ◊ are continuous t-norm and t-conorm. Suppose P(l, m, s) is strictly increasing and Q(l, m, s) is strictly decreasing in s.

$$\lim_{s\to\infty} P(l,m,s) = 1$$

$$\lim_{s\to\infty} Q(l,m,s) = 0$$

If a mapping h: I → I satisfies:

P(hl, hm, s) ≥ P(l, m, s) and Q(hl, hm, s) ≥ Q(l, m, s) for all l, m ∈ I and s ≥ 0,

Then, h has a unique fixed point in I.

Proof: The proof follows directly from the theorem by setting k = 1.

Since P(hl, hm, s) ≥ P(l, m, s) and Q(hl, hm, s) ≥ Q(l, m, s) for all l, m ∈ I and s ≥ 0, we can choose k = 1, which satisfies 0 < k ≤ 1.

By applying the theorem with k = 1, we conclude that h has a unique fixed point in I.

Remark:
This corollary relaxes the contraction condition (ks instead of s) and still guarantees the existence of a unique fixed point for the mapping h.

Theorem 2
Suppose $(I, P, Q,*,\Diamond, b)$ is a complete Intutionistic fuzzy strong b-metric space, * is continuous t-norm defined as * = min{x_1, x_2}, h is a mapping $h: I \to I$ defined by

$$P(hl,hm,ks) \geq P(l,hl,s) * P(m,hm,s) \quad \forall l,m \,\epsilon\, I \;\&\; s \geq 0 \text{ \_\_\_\_\_\_\_\_\_\_\_\_(1)}$$

$$Q(hl,hm,ks) \leq Q(l,hl,s) \Diamond Q(m,hm,s) \quad \forall l,m \,\epsilon\, I \;\&\; s \geq 0 \text{ \_\_\_\_\_\_\_\_\_\_\_\_(2)}$$

Where $0 < k < 1$, $P(l,m,s)$ and $Q(l,m,s)$ are strictly increasing and strictly decreasing in variable s

$$\lim_{s\to\infty} P(l,m,s) = 1 \quad \forall\, l,m \in I \text{ \_\_\_\_\_\_\_\_\_\_\_\_(3)} \lim_{s\to\infty} Q(l,m,s) = 0 \quad \forall\, l,m \in I \text{ \_\_\_\_\_\_\_\_\_\_\_\_(4)}$$

then there exists a unique fixed point of h.

Proof: Consider $q_0 \in I$, then $hq_0 \in I$.
Let $q_1 \in I$ such that $q_1 = hq_0$

By induction, we find a sequence $q_n = hq_{n-1}$ in I.

Now, $P(q_n, q_{n+1}, ks) = P(hq_{n-1}, hq_n, ks)$

$$\geq P(q_{n-1}, hq_{n-1}, s) * P(q_n, hq_n, s)$$

$$\geq P(q_{n-1}, q_n, s) * P(q_n, hq_{n+1}, s)$$

Since $P(l,m,s)$ is strictly increasing in variable s and $ks < s$, so we are unable to write

$$P(q_n, q_{n+1}, ks) \geq P(q_n, q_{n+1}, s) \text{ \_\_\_\_\_\_\_\_\_\_\_\_(5)}$$

Therefore, $P(q_n, q_{n+1}, ks) \geq P(q_{n-1}, q_n, s) = P(hq_{n-2}, hq_{n-1}, s)$

$$\geq P\left(hq_{n-2}, hq_{n-2}, \frac{s}{k}\right) * P\left(hq_{n-1}, hq_{n-1}, \frac{s}{k}\right)$$

$$\geq P\left(hq_{n-2}, hq_{n-1}, \frac{s}{k}\right) * P\left(hq_{n-1}, hq_{n}, \frac{s}{k}\right)$$

$$\geq P\left(hq_0, hq_1, \frac{s}{k^{n-1}}\right)$$

$$P(q_n, q_{n+1}, bs) \geq P\left(hq_0, hq_1, \frac{s}{k^{n-1}}\right) \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_(6)}$$

By using (IFsbM4) and let for every $n \in N$ and positive integer r and $s \geq 0$

$$P(q_n, q_{n+r}, s) \geq P\left(q_n, q_{n+1}, \frac{s}{2}\right) * P\left(q_{n+1}, q_{n+r}, \frac{s}{2b}\right)$$

$$\geq P\left(q_n, q_{n+1}, \frac{s}{2}\right) * P\left(q_{n+1}, q_{n+2}, \frac{s}{4b}\right) * P\left(q_{n+2}, q_{n+r}, \frac{s}{4b^2}\right)$$

$$\geq P\left(q_n, q_{n+1}, \frac{s}{2}\right) * P\left(q_{n+1}, q_{n+2}, \frac{s}{4b}\right) * P\left(q_{n+2}, q_{n+3}, \frac{s}{8b^2}\right) * P\left(q_{n+3}, q_{n+4}, \frac{s}{8b^2}\right) * - - - * P\left(q_{n+r-1}, q_{n+r}, \frac{s}{2^{r-1}b^{r-1}k^{n+r-1}}\right)$$

By using inequality (6)

$$P(q_n, q_{n+r}, s) \geq P\left(q_0, q_1, \frac{s}{2k^n}\right) * P\left(q_0, q_1, \frac{s}{2^2bk^{n+1}}\right) * \ldots * P\left(q_0, q_1, \frac{s}{2^r b^{r-1} k^{n+r-1}}\right)$$

Since $0 < k < 1$. when $n \to \infty$, $k^n \to 0$

$$\lim_{n \to \infty} P(q_n, q_{n+r}, s) \geq 1 * 1 * 1 * \ldots * 1 = 1 \qquad \text{\_\_\_\_\_\_\_\_(7)}$$

Now, $\quad Q(q_n, q_{n+1}, ks) = Q(hq_{n-1,} hq_n, ks)$

$$\leq Q(q_{n-1}, hq_{n-1}, s) \lozenge Q(q_n, hq_n, s)$$

$$\leq Q(q_{n-1}, q_n, s) \lozenge Q(q_n, hq_{n+1}, s)$$

Since $Q(l, m, s)$ is strictly decreasing in variable s and $ks < s$, so we are unable to write

$$Q(q_n, q_{n+1}, ks) \leq Q(q_n, q_{n+1}, s) \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_(8)}$$

Therefore, $\quad Q(q_n, q_{n+1}, ks) \leq Q(q_{n-1}, q_n, s) = Q(hq_{n-2}, hq_{n-1}, s)$

$$\leq Q\left(hq_{n-2}, hq_{n-2}, \frac{s}{k}\right) \lozenge Q\left(hq_{n-1}, hq_{n-1}, \frac{s}{k}\right)$$

$$\leq Q\left(hq_{n-2}, hq_{n-1}, \frac{s}{k}\right) \lozenge Q\left(hq_{n-1}, hq_n, \frac{s}{k}\right)$$

$$\leq Q\left(hq_0, hq_1, \frac{s}{k^{n-1}}\right)$$

$$Q(q_n, q_{n+1}, bs) \leq Q\left(hq_0, hq_1, \frac{s}{k^{n-1}}\right) \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_(9)}$$

By using (IFsbM4) and let for every $n \in N$ and positive integer r and $s \geq 0$

$$Q(q_n, q_{n+r}, s) \leq Q\left(q_n, q_{n+1}, \frac{s}{2}\right) \lozenge Q\left(q_{n+1}, q_{n+r}, \frac{s}{2b}\right)$$

$$\leq Q\left(q_n, q_{n+1}, \frac{s}{2}\right) \lozenge Q\left(q_{n+1}, q_{n+2}, \frac{s}{4b}\right) \lozenge Q\left(q_{n+2}, q_{n+r}, \frac{s}{4b^2}\right)$$

$$\leq Q\left(q_n, q_{n+1}, \frac{s}{2}\right) \lozenge Q\left(q_{n+1}, q_{n+2}, \frac{s}{4b}\right) \lozenge Q\left(q_{n+2}, q_{n+3}, \frac{s}{8b^2}\right) \lozenge Q\left(q_{n+3}, q_{n+4}, \frac{s}{8b^2}\right) \lozenge - - - \lozenge Q\left(q_{n+r-1}, q_{n+r}, \frac{s}{2^{r-1}b^{r-1}k^{n+r-1}}\right)$$

By using inequality (9)

$$Q(q_n, q_{n+r}, s) \leq Q\left(q_0, q_1, \frac{s}{2k^n}\right) \lozenge Q\left(q_0, q_1, \frac{s}{2^2bk^{n+1}}\right) \lozenge \ldots \lozenge Q\left(q_0, q_1, \frac{s}{2^r b^{r-1} k^{n+r-1}}\right)$$

Since $0 < k < 1$. when $n \to \infty$, $k^n \to 0$

$$\lim_{n\to\infty} Q(q_n, q_{n+r}, s) \leq 0 \lozenge 0 \lozenge 0 \lozenge \ldots \lozenge 0 = 0 \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_(10)}$$

Equation (7) and (10) implies $\{q_n\}$ is a Cauchy sequence in I and I is complete, So there exists a point p in I such that

$$\lim_{n\to\infty} q_n = p$$

By using contractive condition

$$P(hq_n, hp, ks) \geq P(q_n, hq_n, s) * P(p, hp, s)$$

$$\geq P(q_n, q_{n+1}, s) * P(p, hp, s)$$

When $n \to \infty$,

$$P(p, hp, ks) \geq P(p, p, s) * P(p, hp, s)$$

$$\geq 1 * P(p, hp, s)$$

$$P(p, hp, ks) \geq P(p, hp, s)$$

By using contractive condition

$$Q(hq_n, hp, ks) \leq Q(q_n, hq_n, s) \lozenge Q(p, hp, s)$$

$$\leq Q(q_n, q_{n+1}, s) \lozenge Q(p, hp, s)$$

When $n \to \infty$,

$$Q(p, hp, ks) \leq Q(p, p, s) \lozenge Q(p, hp, s)$$

$$\leq 0 \lozenge Q(p, hp, s)$$

$$Q(p, hp, ks) \leq Q(p, hp, s)$$

This is a contradiction to the condition $P(l,m,s)$ is strictly increasing and $Q(l,m,s)$ is strictly decreasing in variable s.

Hence, $hp = p$. So, p is the fixed point of mapping h.

Uniqueness: Let p, p* be two fixed points of mapping h, then

$$hp = p \quad \text{and} \quad hp^* = p^*$$

$$P(p,p^*,s) \geq P(hp,hp^*,s)$$

$$\geq P\left(p,hp,\frac{s}{k}\right) * P\left(p^*,hp^*,\frac{s}{k}\right)$$

$$\geq 1 * 1$$

$$P(p,p^*,s) \geq 1$$

$$p = p^*$$

$$Q(p,p^*,s) \leq Q(hp,hp^*,s)$$

$$\leq Q\left(p,hp,\frac{s}{k}\right) \lozenge Q\left(p^*,hp^*,\frac{s}{k}\right)$$

$$\leq 0 \lozenge 0$$

$$Q(p,p^*,s) \leq 0$$

So, $p = p^*$

Then, there exists a unique fixed point of h.

Example:2
Let $I = [0,1]$ and $P: I \times I \times [0,\infty) \to [0,1]$ & $Q: I \times I \times [0,\infty) \to [0,1]$ be defined as

$$P(l,m,s) = \begin{cases} min\{l,m\} + \frac{s}{s+1} & if\ s > 0 \\ min\{l,m\} & if\ s = 0 \end{cases}$$

$$Q(l,m,s) = \begin{cases} \frac{|l-m|}{s+1} & if\ s > 0 \\ |l-m| & if\ s = 0 \end{cases}$$

$h: I \to I$ be defined by $h(x) = kx^2$ and $k = \frac{1}{2}$. * is minimum t-norm and ◊ is t-conorm.

When $s > 0$

$$P(hl,hm,s) = min\{l^2,m^2\} + \frac{s}{s+1}$$

$$\geq \left(\min\{l,l^2\} + \frac{s}{s+1}\right) * \left(\min\{m,m^2\} + \frac{s}{s+1}\right)$$

$$= P(l,hl,s) * P(m,hm,s)$$

$\Rightarrow$ $$P(hl,hm,s) \geq P(l,hl,s) * P(m,hm,s)$$

When $s = 0$

$$P(hl,hm,0) = min\{l^2,m^2\}$$

$$\geq (\min\{l, l^2\}) * (\min\{m, m^2\})$$

$$= P(l, hl, s) * P(m, hm, s)$$

$$\Rightarrow \qquad P(hl, hm, 0) \geq P(l, hl, 0) * P(m, hm, 0)$$

When $s > 0$

$$Q(hl, hm, s) = \frac{|l^2 - m^2|}{s+1}$$

$$\leq \left(\frac{|l - l^2|}{s+1}\right) \lozenge \left(\frac{|m - m^2|}{s+1}\right)$$

$$= Q(l, hl, s) \lozenge Q(m, hm, s)$$

$$\Rightarrow \qquad Q(hl, hm, s) \leq Q(l, hl, s) \lozenge Q(m, hm, s)$$

When $s = 0$

$$Q(hl, hm, 0) = |l^2 - m^2|$$

$$\leq |l - l^2| \lozenge |m - m^2|$$

$$\Rightarrow \qquad Q(hl, hm, 0) \leq Q(l, hl, 0) \lozenge Q(m, hm, 0)$$

Hence, function h has a fixed point.

Example:3

Let $I = [0,1]$ and $P: I \times I \times [0, \infty) \to [0,1]$ & $Q: I \times I \times [0, \infty) \to [0,1]$ be defined as

$$P(l, m, s) = \begin{cases} \dfrac{l+m}{2+s} & if\ s > 0 \\ \dfrac{l+m}{2} & if\ s = 0 \end{cases}$$

$$Q(l, m, s) = \begin{cases} \frac{|l-m|}{2+s} & if\ s > 0 \\ \frac{|l-m|}{2} & if\ s = 0 \end{cases}$$

$h: I \to I$ be defined by $h(x) = \frac{x}{2}$ and $k = \frac{3}{4}$. * is minimum t-norm and $\lozenge$ is t-conorm.

When $s > 0$

$$P(hl, hm, ks) = P\left(hl, hm, \frac{3}{4}s\right)$$

$$= \frac{l/_2 + m/_2}{2+s}$$

$$\geq \left(\frac{l + l/_2}{2+s}\right) * \left(\frac{m + m/_2}{2+s}\right)$$

$$= P(l, hl, s) * P(m, hm, s)$$

$$\Rightarrow \qquad P(hl, hm, ks) \geq P(l, hl, s) * P(m, hm, s)$$

Now, $\qquad Q(hl, hm, ks) = Q\left(hl, hm, \frac{3}{4}s\right)$

$$= \left|\frac{l/_2 - m/_2}{2+s}\right|$$

$$\leq \left(\frac{|l-l/2|}{2+s}\right) \Diamond \left(\frac{|m-m/2|}{2+s}\right)$$

$$= Q(l, hl, s) \Diamond Q(m, hm, s)$$

$$\Rightarrow \quad Q(hl, hm, s) \leq Q(l, hl, s) \Diamond Q(m, hm, s)$$

When $s = 0$

$$P(hl, hm, ks) = P(hl, hm, 0)$$

$$= \frac{l/2 + m/2}{2}$$

$$\geq \left(\frac{l + l/2}{2}\right) * \left(\frac{m + m/2}{2}\right)$$

$$= P(l, hl, 0) * P(m, hm, 0)$$

$$\Rightarrow \quad P(hl, hm, ks) \geq P(l, hl, s) * P(m, hm, s)$$

Now,

$$Q(hl, hm, ks) = Q(hl, hm, 0)$$

$$= \left|\frac{l/2 - m/2}{2}\right|$$

$$\leq \left(\frac{|l-l/2|}{2}\right) \Diamond \left(\frac{|m-m/2|}{2}\right)$$

$$= Q(l, hl, 0) \Diamond Q(m, hm, 0)$$

$$\Rightarrow \quad Q(hl, hm, s) \leq Q(l, hl, s) \Diamond Q(m, hm, s)$$

Hence function h has a fixed point.

Conclusion:

Our findings lay the groundwork for future investigations into the properties and applications of intuitionistic fuzzy strong b-metric spaces. The implications of this study extend beyond the theoretical realm, offering benefits to researchers and practitioners working with uncertain systems.

Specifically, the established fixed point results can be employed to model and analyze various real-world problems involving uncertainty, such as image processing, optimization, and decision-making. In fuzzy mathematics and logic, these findings provide new tools for modeling uncertain systems. Additionally, practitioners in image and signal processing can utilize these results to enhance image segmentation and denoising. Decision-makers operating in fuzzy environments can also leverage these fixed-point results to facilitate optimal solution identification. Further research directions include extending these results to other fuzzy metric spaces, such as intuitionistic fuzzy metric spaces with different underlying structures.

This paper has established the existence of fixed points in Intuitionistic Fuzzy Strong b-Metric Spaces, providing significant contributions to the advancement of fixed point theory in fuzzy metric spaces. The research presents sufficient conditions for the existence of unique fixed points under specific contractive conditions, demonstrated through supportive examples. Intuitionistic Fuzzy b-Metric Space (IFbMS) and Intuitionistic Fuzzy Strong b-Metric Space (IFSbMS) are related but distinct concepts. Every Intuitionistic Fuzzy Strong b-Metric Space (IFSbMS) is an Intuitionistic Fuzzy b-Metric Space (IFbMS), but the converse is not necessarily true. In other words IFSbMS ⊆ IFbMS.

In conclusion, this research contributes meaningfully to the development of fixed point theory in intuitionistic fuzzy strong b-metric spaces, offering valuable insights and tools for theoretical and practical applications. This paper has established the existence of fixed points in Intuitionistic Fuzzy Strong b-Metric Spaces, providing significant contributions to the advancement of fixed point theory in fuzzy metric spaces. The research presents sufficient conditions for the existence of unique fixed points under specific contractive conditions, demonstrated through supportive examples.

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