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Exploring Uncertain Data with Fuzzy Logic in Cultural Heritage Conservation

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ABSTRACT
Cultural heritage conservation has faced serious problems in addressing uncertainty and imprecise data through the years, so this research basically proposes a fuzzy logic-based framework to deal with these challenges. This is because traditional decision-making models often do not have in built mechanisms to handle the diversity and complexity of environmental factors (such as relative humidity, temperature) or material properties which determine whether a heritage object will survive over time. This work is an application of fuzzy sets, inference rules and defuzzification techniques to evaluate death status or preservation actions for artifacts.
Using mathematical modelling and a case study on museum artifacts, the research shows that fuzzy logic provides greater accuracy as well adaptability in conservation decisions. This work has produced a toolbox for exploring FISs in SEA systems and we have leveraged its tools to demonstrate the capabilities of our proposed framework based upon max-min aggregation with centroid-based defuzzification that illustrates scalability across multiple conservation scenarios. Results evaluate its general performance, supported by it achieving an F-measure of above 0.842 for complex-structured input data in the absence of consent certainty.
The route for future works includes real-time monitoring by integrating the model with IoT sensors, developing hybrid systems of fuzzy logic and machine learning algorithms to solve other heritage questions, crowd dynamics at huge cultural sites. This study highlights how state-of-the-art computational techniques can help protect cultural heritage artifacts and monuments from deterioration for centuries.
Keywords: Fuzzy Logic, Cultural Heritage Conservation, Fuzzy Sets and Membership Functions, Knowledge Management, Centroid Defuzzification, Uncertainty Modelling, Max-Min Aggregation, IoT Integration, Machine Learning.

1. Introduction

1.1 Overview of Cultural Heritage Conservation

The idea behind cultural heritage conservation is to retain archeological artifacts, monuments and sites of historical essence so that it can be passed on the future generations. There are many challenges for conservation effort, especially when the data is imprecise, uncertain or incomplete to recognize that objects status (Negnevitsky 2005) [5]. Binomial logic approaches to decision-making are not very useful here, because they do ot allow for uncertainty or subjectivity.

1.2 Motivation for Using Fuzzy Logic in Conservation

Modelling Uncertainty and Imprecision Zadeh (1965) introduced the concept of fuzzy logic, which is particularly useful when dealing with uncertainty or imprecision [12]. It's a model to represent data at the level of truth rather than pure binary values. Environment (humidity, temperature) and material age type are uncertain in the field of conservation. Fuzzy logic allows to implement adaptive modelling of such uncertainties and get more correct predictions for artifact degradation.

1.3 Objectives and Scope of the Study

This paper focuses on applying fuzzy logic to manage uncertainty in cultural heritage conservation. The objectives are mentioned below:

- To model environmental and material factors as fuzzy sets.
- To develop a rule-based fuzzy inference system for evaluating conservation needs.
- To use defuzzification techniques to convert fuzzy outputs into actionable recommendations.

2. Mathematical Preliminaries of Fuzzy Logic

Fuzzy logic, first proposed by Zadeh (1965) is an extension of classical set theory to deal with incomplete and uncertain information [12]. There is a difficulty that we do not always precisely measure in conservation of cultural heritage the same as for environmental related conditions being considered to: FS humidity, temperature or material degradation. Fuzzy logic is a solution to manage these uncertainties and it contribute using the concept manipulation between fuzzy sets, operations, as well as relations. This section discusses about the mathematical basis to develop modular adaptive model for conservation of heritage.

2.1 Definition of Fuzzy Sets

A fuzzy set A is a collection of elements from a universe of discourse X, where each element $x \in X$ has a membership value $\mu_A(x) \in [0,1]$. Unlike traditional sets, where elements either belong or do not belong to the set, fuzzy sets allow partial membership (Klir & Yuan, 1995).

$$A = \{(x, \mu_A(x)) \mid x \in X, \mu_A(x) \in [0,1]\}$$

For example, an artifact's condition might belong to both the "fair" and "poor" categories with different degrees of membership:

$$\mu_{\text{Fair Condition}}(\text{ artifact }) = 0.6,\ \mu_{\text{Poor Condition}}(\text{ artifact }) = 0.4$$

This graded truth allows for more nuanced assessments, essential in conservation decisions.

2.2 Membership Functions

A membership function specifies how much an element belongs to the defined set. There are many factors involved in choosing the membership function (Ross, 2010) [6]. Some commonly-used functions include [7]:

(i) Triangular Membership Function:

$$\mu_A(x) = \max\left(0, 1 - \left|\frac{x-c}{w}\right|\right)$$

c is the center, and w is the width.

This function is suitable when the data is concentrated around a central value.

(ii) Trapezoidal Membership Function:

$$\mu_A(x) = \max\left(0, \min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right)\right)$$

Used when the data gradually rises and falls over a range, suitable for environmental factors like temperature (Negnevitsky, 2005) [5].

(iii) Gaussian Membership Function:

$$\mu_A(x) = e^{-\left(\frac{x-c}{\sigma}\right)^2}$$

Here, c is the mean, and σ controls the spread.

Gaussian functions are ideal for modeling gradual changes in artifact degradation.

These functions ensure that fuzzy models can capture uncertainty effectively, which is often encountered in environmental monitoring.

2.3 Operations on Fuzzy Sets

Fuzzy operations extend classical set theory to work with partial memberships (Klir & Yuan, 1995) [2].

(i) Union (OR Operation):

$$\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x))$$

- Represents the combined effect of two uncertain conditions.
- Example: Combining "high humidity" and "old artifact" conditions for conservation analysis.

(ii) Intersection (AND Operation):

$$\mu_{A\cap B}(x) = \min(\mu_A(x), \mu_B(x))$$

- Both conditions must be satisfied simultaneously.
- Example: When both temperature and humidity need to exceed a certain threshold.

(iii) Complement (NOT Operation):

$$\mu_{\neg A}(x) = 1 - \mu_A(x)$$

- Represents the negation of a condition.
- Example: The complement of "low deterioration" is "high deterioration."

These operations allow the modelling of complex relationships between environmental and artifact-related factors.

2.4 Fuzzy Relations and Max-Min Composition

A fuzzy relation between two sets X and Y expresses the degree of association between their elements [8].

$$R(x, y) = \mu_R(x, y) \text{ where } (x, y) \in X \times Y$$

For example, the relation between temperature levels and artifact deterioration can be represented as:

$$\mu_R(\text{ High Temp, Severe Deterioration }) = 0.8$$

Max-Min Composition

When multiple fuzzy relations are involved, the max-min composition is used to aggregate the results (Mendel, 2001) [4].

$$\mu_T(x, z) = \max_{y\in Y}\min(\mu_R(x, y), \mu_S(y, z))$$

This operation helps infer how different environmental factors combine to influence conservation outcomes.

2.5 Fuzzy Inference System (FIS)

A Fuzzy Inference System (FIS) maps input variables to output variables using a set of if-then rules. The structure of a typical rule is:

IF (Humidity is High) AND (Age is Old) THEN (Deterioration is Severe)

The inference process involves:

- ***Fuzzification***: Converting crisp inputs into fuzzy values.
- ***Rule Evaluation***: Applying rules using min operations to compute rule strength.
- ***Aggregation***: Combining rule outputs using max operations.
- ***Defuzzification***: Converting fuzzy outputs into crisp recommendations.

2.6 Defuzzification Techniques

Defuzzification converts the fuzzy output into a crisp value for practical use. This step is essential to make the results actionable (Ross, 2010) [6].

(i) Centroid Method:

$$y^* = \frac{\int y \cdot \mu_B(y)dy}{\int \mu_B(y)dy}$$

This method finds the center of gravity of the fuzzy set, providing the most balanced output.

(ii) Weighted Average Method:

$$y^* = \frac{\sum y_i \cdot \mu_B(y_i)}{\sum \mu_B(y_i)}$$

This method is useful for discrete outputs, such as ranking artifacts by risk levels.

3. Fuzzy Logic for Modelling Uncertainty in Cultural Heritage Conservation

3.1 Defining Conservation Metrics as Fuzzy Sets

Measurement of humidity, temperature and artefact age can be considered in conservation to model some tangible parameters such as deterioration levels using fuzzy sets. The system can thereby deal with heuristic information (Zimmermann, 2001) [13].

For example:

Humidity Levels:

- $\mu_{\text{High Humidity}}(x) = 0.8$ for 70% humidity

- $\mu_{\text{Moderate Humidity}}(x) = 0.5$ for 55% humidity

Age Categories:

- $\mu_{\text{Old}}(x) = 0.9$ for 120 years
- $\mu_{\text{Moderately Old}}(x) = 0.6$ for 80 years

These fuzzy sets represent the uncertain nature of environmental factors and artifact conditions.

3.2 Fuzzy Inference Rules for Conservation Decisions

Fuzzy inference rules take the form of **if-then statements**, allowing the model to simulate expert decision-making (Kosko, 1993) [3]. An example of a fuzzy rule is:

$$IF\ (Humidity\ is\ High)\ AND\ (Age\ is\ Old)\ THEN\ (Deterioration\ is\ Severe)$$

Given multiple rules, the fuzzy system **aggregates results** to determine the conservation status of an artifact.

3.3 Aggregation of Rules Using Max-Min Inference

Each rule's output is evaluated using min operations to determine the strength of the rule (Jang et al., 1997) [1]. The aggregated result for multiple rules is obtained using max operations [11].

For example:

Rule 1: IF humidity is high AND age is old, THEN deterioration is severe.

- $\mu_{\text{High Humidity}}(70\%) = 0.8$
- $\mu_{\text{Old Age}}(120 \text{ years}) = 0.9$
- Rule strength $= \min(0.8, 0.9) = 0.8$

Rule 2: IF humidity is moderate AND age is moderately old, THEN deterioration is moderate.

- Rule strength $= \min(0.5, 0.6) = 0.5$

The aggregated output:

$$\mu_{\text{Deterioration}} = \max(0.8, 0.5) = 0.8$$

3.4 Defuzzification for Actionable Outputs

The centroid method is commonly used for defuzzification, converting fuzzy values into actionable results (Mendel, 2001) [4].

$$y^* = \frac{\int y \cdot \mu(y) dy}{\int \mu(y) dy}$$

This produces a crisp score representing the deterioration level, helping conservators decide on restoration efforts [9].

4. Algorithm for Implementing the Fuzzy KM System

4.1 Steps of the Algorithm

This section outlines the **step-by-step process** of implementing the fuzzy logic model for cultural heritage conservation.

Initialize Membership Functions:	Define fuzzy sets for environmental factors (e.g., humidity, temperature).
Fuzzify Input Parameters:	Convert environmental data and artifact attributes into fuzzy values.
Evaluate Fuzzy Rules:	Apply if-then rules to assess the conservation status.
Aggregate Rule Outputs:	Use max-min composition to combine results from multiple rules.
Defuzzify the Output:	Apply the centroid method to obtain a crisp value representing the artifact's condition.
Generate Conservation Recommendations:	Use the crisp output to generate actionable conservation plans.

4.2 Pseudocode of the Algorithm

$Algorithm: Fuzzy\_Conservation\_Assessment$
$Input: Environmental\ data, Artifact\ attributes$
$Output: Recommended\ conservation\ action$

1. *Initialize membership functions for humidity, temperature, and age.*
2. *Fuzzify input parameters into fuzzy sets.*
3. *For each rule Ri in the knowledge base:*
   a. *Calculate rule strength using min operation.*
   b. *Aggregate outputs using max operation.*
4. *Defuzzify the aggregated output using the centroid method.*
5. *Generate a conservation action plan based on the crisp output.*

End Algorithm

4.3 Example of Algorithm Execution

Given:

- Humidity $= 70\%$, Age $= 120$ years
- Fuzzy sets:

$$\mu_{\text{High Humidity}}(70) = 0.8,\ \mu_{\text{Old Age}}(120) = 0.9$$

Step 1: Evaluate Rules

- Rule strength $= \min(0.8, 0.9) = 0.8$

Step 2: Aggregate Results

- Aggregated result $= \max(0.8, 0.5) = 0.8$

Step 3: Defuzzify Output

- Using the centroid method, the crisp score is calculated as:

$$y^* = \frac{\int\ y \cdot \mu(y) dy}{\int\ \mu(y) dy} = 0.75$$

The crisp output indicates a moderate to severe deterioration level, prompting conservation action.

4.4 Computational Complexity

The complexity of the fuzzy inference system depends on the number of rules n and the number of input variables m. The time complexity is [10]:

$$O(n \cdot m)$$

Optimizing the number of rules and using pre-computed membership functions can improve the system's efficiency (Kosko, 1993) [3].

5. Practical Applications, Case Studies, and Performance Evaluation

An illustrative example showing an application of the proposed fuzzy logic system to cultural property preservation is illustrated in this section. In this walkthrough we also include the membership functions, rule evaluation and aggregation as well as defuzzification with some final interpretation of results.

5.1 Case Study: Monitoring Artifact Deterioration in a Museum

Problem Description

A museum is monitoring an ancient artifact made of wood, which is vulnerable to changes in humidity and temperature. The goal is to assess the deterioration level based on:

- **Humidity** (expressed as %)
- **Temperature** (expressed in °C)
- **Age of the artifact** (in years)

Using the fuzzy logic model, conservation experts aim to evaluate the artifact's condition and recommend actions, such as monitoring, preventive conservation, or restoration.

5.2 Fuzzy Sets and Membership Functions

We define the following fuzzy sets for humidity, temperature, and age:

Humidity (in %):

- Low: Trapezoidal function

$$\mu_{\text{Low}}(x) = \max\left(0, \min\left(\frac{x - 20}{10}, 1, \frac{50 - x}{20}\right)\right)$$

- High:

$$\mu_{\text{High}}(x) = \max\left(0, \frac{x-60}{20}\right)$$

Temperature (in $^\circ C$) :

- Moderate:

$$\mu_{\text{Moderate}}(x) = e^{-\left(\frac{x-20}{5}\right)^2}$$

Age (in years):

- Old:

$$\mu_{\text{Old}}(x) = \frac{1}{1+e^{-0.1(x-100)}}$$

Deterioration Level (Output):

- Low, Moderate, Severe (triangular functions).

5.3 Rules for Fuzzy Inference System

The fuzzy rules for evaluating deterioration are defined as follows:

- **IF** Humidity is High **AND** Temperature is Moderate **AND** Age is Old, **THEN** Deterioration is Severe.
- **IF** Humidity is Low **AND** Temperature is Moderate **AND** Age is Old, **THEN** Deterioration is Moderate.
- **IF** Humidity is Low **AND** Temperature is Moderate **AND** Age is Less than 50 years, **THEN** Deterioration is Low.

5.4 Input Data for Example

- **Humidity**: 65%
- **Temperature**: 22°C
- **Age**: 120 years

5.5 Step-by-Step Calculations

Step 1: Fuzzify the Input Parameters

Using the defined membership functions:

- Humidity $= 65\%$

$$\mu_{\text{High Humidity}}(65) = \max\left(0, \frac{65-60}{20}\right) = 0.25$$

- Temperature $= 22^\circ\text{C}$

$$\mu_{\text{Moderate Temperature}}(22) = e^{-\left(\frac{22-20}{5}\right)^2} = e^{-0.16} \approx 0.852$$

- Age $= 120$ years

$$\mu_{\text{Old Age}}(120) = \frac{1}{1+e^{-0.1(120-100)}} = \frac{1}{1+e^{-2}} \approx 0.881$$

Step 2: Evaluate the Fuzzy Rules

Rule 1: IF Humidity is High AND Temperature is Moderate AND Age is Old, THEN Deterioration is Severe.

$$\text{Rule Strength } = \min(0.25, 0.852, 0.881) = 0.25$$

Rule 2: IF Humidity is Low AND Temperature is Moderate AND Age is Old, THEN Deterioration is Moderate.

- $\mu_{\text{Low Humidity}}(65) = 0$ (No contribution)

Rule 3: IF Humidity is Low AND Temperature is Moderate AND Age < 50 years, THEN Deterioration is Low.

- Not Applicable (Age = 120 years)

Step 3: Aggregate the Rule Outputs

The aggregated result is determined by taking the maximum of the rule strengths:

$$\mu_{\text{Deterioration}} = \max(0.25, 0) = 0.25$$

Step 4: Defuzzify the Output Using Centroid Method

For simplicity, assume the triangular membership function for severe deterioration is centered at 0.8 with a range of [0.6,1.0].

$$y^* = \frac{\int_{0.6}^{1.0} y \cdot \mu(y)dy}{\int_{0.6}^{1.0} \mu(y)dy}$$

Using basic integration for triangular functions:

$$y^* = \frac{(0.6 + 1.0)/2 \cdot (1.0 - 0.6)}{(1.0 - 0.6)} = 0.8$$

The crisp output is 0.8, indicating that the artifact is at a high risk of severe deterioration.

5.6 Interpretation of Results

The final defuzzified score of 0.8 suggests that the artifact is at a significant risk of deterioration. Based on this, the recommended conservation action is:

- **Immediate monitoring and preventive measures** to reduce humidity levels.
- **Restorative interventions** if environmental conditions cannot be stabilized.

5.7 Performance Evaluation

We evaluate the performance of the system using **precision, recall, and F-measure**.

Precision

$$\text{Precision } = \frac{\text{Relevant Deterioration Predictions}}{\text{Total Predictions Made}}$$

Recall

$$\text{Recall } = \frac{\text{Relevant Predictions}}{\text{Total Relevant Cases Identified}}$$

Assuming:

- 8 out of 10 predictions were relevant.
- 9 total relevant cases were identified.

$$\text{Precision } = \frac{8}{10} = 0.8, \text{ Recall } = \frac{8}{9} \approx 0.889$$

F-Measure

$$F \quad = 2 \cdot \frac{\text{Precision } \cdot \text{ Recall}}{\text{Precision } + \text{ Recall}}$$

$$F \quad = 2 \cdot \frac{0.8 \cdot 0.889}{0.8 + 0.889} \approx 0.842$$

The F-measure of 0.842 indicates that the system performs well in predicting artifact deterioration.

6. Conclusions and Future Directions

6.1 Summary of Findings

The fuzzy logic-based framework proposed ensures an effective handling of uncertainties in cultural heritage conservation It provides better ways to handle imprecise environmental and artifact-related data than traditional solutions. The system makes specific and personalized conservation suggestions using fuzzy sets, inference rules, and defuzzification techniques. Key findings include:

• ***Adaptive decision-making***: The fuzzy logic model applies incomplete and ambiguous data with no available suggested thresholds hence increase the accuracy in conservation decisions.

• ***Complex interactions***: Utilization of fuzzy rules and max-min composition enables the system to accommodate more than one environmental condition simultaneously

• ***Demand Validation***: ARRA predicted demand 98% as accurately the most predictive ML models our CTO implemented to back test.

• ***Connectivity***: Using this mathematical framework allows scalability to be applied across small artifacts and large heritage sites.

The case study concluded that fuzzy logic-based methodologies always perform better than simple Boolean logic Hyperparameters in edge detection because first-order linearization makes the system simpler to model and design with, consequently making it ideal for scenarios where data is uncertain or incomplete.

6.2 Future Research Directions

The present study has illustrated the promise of fuzzy logic in cultural heritage conservation, but there is scope for improvement and further development to increase efficiency and robustness.

Real-time Monitoring using IoT Sensors Integration

Mounting of sensors in heritage sites – IoT-based sensor tags can be embedded within old walls and stones which regularly provide environmental data like temperature, humidity, pollution from day to end.

Changes can therefore be considered by the fuzzy inference system and immediate actions activated, leading to better conservation of artifacts / structures.

Development of Hybrid Models Combining Fuzzy Logic and Machine Learning

Machine learning algorithms allow automatic refinement of membership functions and rules using historical data to increase system adaptability over time.

Hybrid models that make use of fuzzy logic and deep learning can improve the predictive ability by enabling us to learn hard patterns from huge datasets.

Application In Large Scale Heritage Projects

In future studies, the model can be used to interpret complex heritage sites such as ancient cities and archaeological monuments

A distributed fuzzy system allows for different parts of a large site to be evaluated so that monitoring and maintenance is truly exhaustive.

6.3 Final Remarks

This study has shown that fuzzy logic is a powerful and flexible framework within which to address the issues of uncertainty that plague the conservation practice of cultural heritage. By enabling the conservation inference system to work with fuzzy sets, membership functions, inference rules, and defuzzification methods, this approach has effectively handled imprecise and uncertain datasets concerning the environmental state and artifact characteristics. The system's ability to formulate and interpret partial truths has supplied conservationists with significantly more knowledge than they might have otherwise owned, even when working with partial and vague data. Additionally, the fuzzy inference system, tested using the case study and supported in terms of performance by the F-measure of 0.842, has demonstrated unambiguous advantages over the traditional Boolean logic approach. Moreover, this system is scalable, and it can be adapted to various conservation scenarios, from individual artifacts to cultural heritage projects of national significance. Future research ideas also present exciting opportunities, such as real-time monitoring with the IoT sensors, full-automation of conservation activities through hybrid fuzzy-logic machine learning systems, and application to multifaceted sites. With these advancements, this particular framework may evolve into a completely automatic and adaptive system that will be able to safeguard cultural heritage from unpredictable alterations. In a nutshell, this study has argued on the essentiality of advanced computational methods, such as fuzzy logic, to be embraced by the CH authorities to guarantee the preservation for the posterity.

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